**Ormiston Rivers Academy**

**Bridging the gap between GCSE and A Level Mathematics**

**2025-26**

“Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. You go into the first room and it's dark, completely dark. You stumble around, bumping into the furniture. Gradually, you learn where each piece of furniture is. And finally, after six months or so, you find the light switch and turn it on. Suddenly, it's all illuminated and you can see exactly where you were. Then you enter the next dark room…”

Sir Andrew Wiles

A close-up of a math problem

AI-generated content may be incorrect.

A blue text on a white background

AI-generated content may be incorrect.

Introduction

A level Mathematics is a step up from GCSE – the aim of this booklet is to help you with this transition.

Attempt all the questions in each section and check your own answers. Demonstrating working out is an important part of mathematics, in fact it is more important than the final answer. Think about how you set out your answers – will other people (i.e. an examiner) be able to understand your work?

Independent Learning & Resources

An important part of A-levels is that students should carry out independent learning. Independent learners are able to take control of their own learning, they are able to access and choose resources away from the classroom and use this to guide their own route through the subject.

Useful websites:

<https://physicsandmathstutor.co.uk/>

<https://physicsfactory.co.uk/>

<https://www.youtube.com/@MathsWithDan>

<https://www.youtube.com/@BicenMaths>

**Expanding brackets   
and simplifying expressions**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
* When you expand two linear expressions, each with two terms of the form *ax* + *b*, where *a*≠ 0 and *b*≠ 0, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

**Example 1** Expand 4(3*x* − 2)

|  |  |
| --- | --- |
| 4(3*x* − 2) = 12*x* − 8 | Multiply everything inside the bracket by the 4 outside the bracket |

**Example 2** Expand and simplify 3(*x* + 5) − 4(2*x* + 3)

|  |  |
| --- | --- |
| 3(*x* + 5) − 4(2*x* + 3)  = 3*x* + 15 − 8*x* – 12  = 3 − 5*x* | **1** Expand each set of brackets separately by multiplying (*x* + 5) by 3 and (2*x* + 3) by −4  **2** Simplify by collecting like terms: 3*x*− 8*x*= −5*x* and 15 − 12 = 3 |

**Example 3** Expand and simplify (*x* + 3)(*x* + 2)

|  |  |
| --- | --- |
| (*x* + 3)(*x* + 2)  = *x*(*x* + 2) + 3(*x* + 2)  = *x*2 + 2*x* + 3*x* + 6  = *x*2 + 5*x* + 6 | **1** Expand the brackets by multiplying (*x* + 2) by *x* and (*x* + 2) by 3  **2** Simplify by collecting like terms: 2*x*+ 3*x* = 5*x* |

**Example 4** Expand and simplify (*x* − 5)(2*x* + 3)

|  |  |
| --- | --- |
| (*x* − 5)(2*x* + 3)  = *x*(2*x* + 3) − 5(2*x* + 3)  = 2*x*2 + 3*x* − 10*x* − 15  = 2*x*2 − 7*x* − 15 | **1** Expand the brackets by multiplying (2*x* + 3) by *x* and (2*x* + 3) by −5  **2** Simplify by collecting like terms: 3*x*− 10*x* = −7*x* |

Practice

**1** Expand.

**Watch out!**

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is ‘+’; if the signs are different the answer is ‘–’.

**a** 3(2*x* − 1) **b** −2(5*pq* + 4*q*2)

**c** −(3*xy* − 2*y*2)

**2** Expand and simplify.

**a** 7(3*x* + 5) + 6(2*x* – 8) **b** 8(5*p* – 2) – 3(4*p* + 9)

**c** 9(3*s* + 1) –5(6*s* – 10) **d** 2(4*x* – 3) – (3*x* + 5)

**3** Expand.

**a** 3*x*(4*x* + 8) **b** 4*k*(5*k*2 – 12)

**c** –2*h*(6*h*2 + 11*h* – 5) **d** –3*s*(4*s*2 – 7*s* + 2)

**4** Expand and simplify.

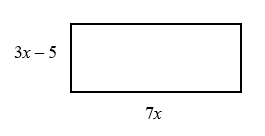
**a** 3(*y*2 – 8) – 4(*y*2 – 5) **b** 2*x*(*x* + 5) + 3*x*(*x* – 7)

**c** 4*p*(2*p* – 1) – 3*p*(5*p* – 2) **d** 3*b*(4*b* – 3) – *b*(6*b* – 9)

**5** Expand (2*y* – 8)

**6** Expand and simplify.

**a** 13 – 2(*m* + 7) **b** 5*p*(*p*2 + 6*p*) – 9*p*(2*p* – 3)

**7** The diagram shows a rectangle.

Write down an expression, in terms of *x*, for the area of the rectangle.

Show that the area of the rectangle can be written as 21*x*2– 35*x*

**8** Expand and simplify.

**a** (*x* + 4)(*x* + 5) **b** (*x* + 7)(*x* + 3)

**c** (*x* + 7)(*x* – 2) **d** (*x* + 5)(*x* – 5)

**e** (2*x* + 3)(*x* – 1) **f** (3*x* – 2)(2*x* + 1)

**g** (5*x* – 3)(2*x* – 5) **h** (3*x* – 2)(7 + 4*x*)

**i** (3*x* + 4*y*)(5*y* + 6*x*) **j** (*x* + 5)2

**k** (2*x* − 7)2 **l** (4*x* − 3*y*)2

Extend

**9** Expand and simplify (*x* + 3)² + (*x* − 4)²

**10** Expand and simplify.

**a**  **b** 

Answers

**1 a** 6*x* – 3 **b** –10*pq* – 8*q*2

**c** –3*xy* + 2*y*2

**2 a** 21*x* + 35 + 12*x* – 48 = 33*x* – 13

**b** 40*p* – 16 – 12*p* – 27 = 28*p* – 43

**c** 27*s* + 9 – 30*s* + 50 = –3*s* + 59 = 59 – 3*s*

**d** 8*x* – 6 – 3*x* – 5 = 5*x* – 11

**3 a** 12*x*2 + 24*x* **b** 20*k*3 – 48*k*

**c** 10*h* – 12*h*3 – 22*h*2 **d** 21*s*2 – 21*s*3 – 6*s*

**4 a** –*y*2 – 4 **b** 5*x*2 – 11*x*

**c** 2*p* – 7*p*2 **d** 6*b*2

**5** *y* – 4

**6 a** –1 – 2*m* **b** 5*p*3 + 12*p*2 + 27*p*

**7** 7*x*(3*x* – 5) = 21*x*2 – 35*x*

**8 a** *x*2 + 9*x* + 20 **b** *x*2 + 10*x* + 21

**c** *x*2 + 5*x* – 14 **d** *x*2 – 25

**e** 2*x*2 + *x* – 3 **f** 6*x*2 – *x* – 2

**g** 10*x*2 – 31*x* + 15 **h** 12*x*2 + 13*x* – 14

**i** 18*x*2 + 39*xy* + 20*y*2 **j** *x*2 + 10*x* + 25

**k** 4*x*2 − 28*x* + 49 **l** 16*x*2 − 24*xy* + 9*y*2

**9** 2*x*2 − 2*x* + 25

**10 a**  **b** 

**Surds and rationalising the denominator**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* A surd is the square root of a number that is not a square number,   
  for example  etc.
* Surds can be used to give the exact value for an answer.
* 
* 
* To rationalise the denominator means to remove the surd from the denominator of a fraction.
* To rationalise you multiply the numerator and denominator by the surd 
* To rationalise  you multiply the numerator and denominator by 

Examples

**Example 1** Simplify 

|  |  |
| --- | --- |
|  | **1** Choose two numbers that are factors of 50. One of the factors must be a square number  **2** Use the rule  **3** Use |

**Example 2** Simplify 

|  |  |
| --- | --- |
|  | **1** Simplify  and . Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number  **2** Use the rule  **3** Use  and  **4** Collect like terms |

**Example 3** Simplify 

|  |  |
| --- | --- |
| =  = 7 – 2  = 5 | **1** Expand the brackets. A common mistake here is to write  **2** Collect like terms: |

**Example 4** Rationalise 

|  |  |
| --- | --- |
| =  =  = | **1** Multiply the numerator and denominator by  **2** Use |

**Example 5** Rationalise and simplify 

|  |  |
| --- | --- |
| =  =  =  = | **1** Multiply the numerator and denominator by  **2** Simplify  in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number  **3** Use the rule  **4** Use  **5** Simplify the fraction:  simplifies to |

**Example 6** Rationalise and simplify 

|  |  |
| --- | --- |
| =  =  =  =  = | **1** Multiply the numerator and denominator by  **2** Expand the brackets  **3** Simplify the fraction  **4** Divide the numerator by −1  Remember to change the sign of all terms when dividing by −1 |

Practice

**1** Simplify.

**Hint**

One of the two numbers you choose at the start must be a square number.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**2** Simplify.

**Watch out!**

Check you have chosen the highest square number at the start.

**a**  **b** 

**c**  **d** 

**e  f** 

**3** Expand and simplify.

**a**  **b** 

**c**  **d** 

**4** Rationalise and simplify, if possible.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**5** Rationalise and simplify.

**a**  **b**  **c** 

# **Extend**

**6** Expand and simplify 

**7** Rationalise and simplify, if possible.

**a**  **b** 

Answers

**1 a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**2 a**  **b** 

**c**  **d** 

**e**  **f** 

**3 a** −1 **b** 

**c**  **d** 

**4 a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**5 a**  **b**  **c** 

**6** *x* − *y*

**7 a**  **b** 

**Rules of indices**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

**Key points**

* *am* × *an* = *am* + *n*
* 
* (*am*)*n* = *amn*
* *a*0 = 1
*  i.e. the *n*th root of *a*
* 
* 
* The square root of a number produces two solutions, e.g. .

**Examples**

**Example 1** Evaluate 100

|  |  |
| --- | --- |
| 100 = 1 | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate 

|  |  |
| --- | --- |
| = 3 | Use the rule |

**Example 3** Evaluate 

|  |  |
| --- | --- |
| =  = 9 | **1** Use the rule  **2** Use |

**Example 4** Evaluate 

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use |

**Example 5** Simplify 

|  |  |
| --- | --- |
| = 3*x*3 | 6 ÷ 2 = 3 and use the rule  to give |

**Example 6** Simplify 

|  |  |
| --- | --- |
| = *x*8 − 4 = *x*4 | **1** Use the rule  **2** Use the rule |

**Example 7** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | Use the rule , note that the fraction  remains unchanged |

**Example 8** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use the rule |

**Practice**

**1** Evaluate.

**a** 140 **b** 30 **c** 50 **d** *x*0

**2** Evaluate.

**a**  **b**  **c**  **d** 

**3** Evaluate.

**a**  **b**  **c**  **d** 

**4** Evaluate.

**a** 5–2 **b** 4–3 **c** 2–5 **d** 6–2

**5** Simplify.

**a**  **b** 

**c**  **d** 

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule *a*0 = 1.

**e**  **f** 

**g**  **h** 

**6** Evaluate.

**a**  **b**  **c** 

**d**  **e**  **f** 

**7** Write the following as a single power of *x*.

**a**  **b**  **c** 

**d**  **e**  **f** 

**8** Write the following without negative or fractional powers.

**a**  **b** *x*0 **c** 

**d**  **e**  **f** 

**9** Write the following in the form *axn*.

**a**  **b**  **c** 

**d**  **e**  **f** 3

**Extend**

**10** Write as sums of powers of *x*.

**a**  **b**  **c** 

**Answers**

**1 a** 1 **b** 1 **c** 1 **d** 1

**2 a** 7 **b** 4 **c** 5 **d** 2

**3 a** 125 **b** 32 **c** 343 **d** 8

**4 a**  **b**  **c**  **d** 

**5 a**  **b** 5*x*2

**c** 3*x* **d** 

**e**  **f** *c*–3

**g** 2*x*6 **h** *x*

**6 a**  **b**  **c** 

**d**  **e**  **f** 

**7 a** *x*–1 **b** *x*–7 **c** 

**d**  **e**  **f** 

**8 a**  **b** 1 **c** 

**d**  **e**  **f** 

**9 a**  **b** 2*x*–3 **c** 

**d**  **e**  **f** 3*x*0

**10 a**  **b**  **c** 

**Factorising expressions**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

**Key points**

* Factorising an expression is the opposite of expanding the brackets.
* A quadratic expression is in the form *ax*2 + *bx* + *c*, where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose product is *ac*.
* An expression in the form *x*2 – *y*2 is called the difference of two squares. It factorises to (*x* – *y*)(*x* + *y*).

**Examples**

**Example 1** Factorise 15*x*2*y*3 + 9*x*4*y*

|  |  |
| --- | --- |
| 15*x*2*y*3 + 9*x*4*y* = 3*x*2*y*(5*y*2 + 3*x*2) | The highest common factor is 3*x*2*y*. So take 3*x*2*y* outside the brackets and then divide each term by 3*x*2*y* to find the terms in the brackets |

**Example 2** Factorise 4*x*2 – 25*y*2

|  |  |
| --- | --- |
| 4*x*2 – 25*y*2 = (2*x* + 5*y*)(2*x* − 5*y*) | This is the difference of two squares as the two terms can be written as (2*x*)2and (5*y*)2 |

**Example 3** Factorise *x*2 + 3*x* – 10

|  |  |
| --- | --- |
| *b* = 3, *ac* = −10  So *x*2 + 3*x* – 10 = *x*2 + 5*x* – 2*x* – 10  = *x*(*x* + 5) – 2(*x* + 5)  = (*x* + 5)(*x* – 2) | **1** Work out the two factors of *ac*= −10 which add to give *b*= 3  (5 and −2)  **2** Rewrite the *b* term (3*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (*x* + 5) is a factor of both terms |

**Example 4** Factorise 6*x*2 − 11*x* − 10

|  |  |
| --- | --- |
| *b* = −11, *ac* = −60  So  6*x*2 − 11*x* – 10 =6*x*2 − 15*x* + 4*x* – 10  = 3*x*(2*x* − 5) + 2(2*x* − 5)  = (2*x* – 5)(3*x* + 2) | **1** Work out the two factors of *ac*= −60 which add to give *b*= −11 (−15 and 4)  **2** Rewrite the *b* term (−11*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (2*x* − 5) is a factor of both terms |

**Example 5** Simplify 

|  |  |
| --- | --- |
| For the numerator:  *b* = −4, *ac* = −21  So  *x*2 − 4*x* – 21 = *x*2 − 7*x* + 3*x* – 21  = *x*(*x* − 7) + 3(*x* − 7)  = (*x* – 7)(*x* + 3)  For the denominator:  *b* = 9, *ac* = 18  So  2*x*2 + 9*x* + 9 = 2*x*2 + 6*x* + 3*x* + 9  = 2*x*(*x* + 3) + 3(*x* + 3)  = (*x* + 3)(2*x* + 3)  So    = | **1** Factorise the numerator and the denominator  **2** Work out the two factors of *ac*= −21 which add to give *b*= −4 (−7 and 3)  **3** Rewrite the *b* term (−4*x*) using these two factors  **4** Factorise the first two terms and the last two terms  **5** (*x* − 7) is a factor of both terms  **6** Work out the two factors of  *ac*= 18 which add to give *b*= 9  (6 and 3)  **7** Rewrite the *b* term (9*x*) using these two factors  **8** Factorise the first two terms and the last two terms  **9** (*x* + 3) is a factor of both terms  **10** (*x* + 3) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

**Practice**

**1** Factorise.

**Hint**

Take the highest common factor outside the bracket.

**a** 6*x*4*y*3 – 10*x*3*y*4 **b** 21*a*3*b*5 + 35*a*5*b*2

**c** 25*x*2*y*2 – 10*x*3*y*2 + 15*x*2*y*3

**2** Factorise

**a** *x*2 + 7*x* + 12 **b** *x*2 + 5*x* – 14

**c** *x*2 – 11*x* + 30 **d** *x*2 – 5*x* – 24

**e** *x*2 – 7*x* – 18 **f** *x*2 + *x* –20

**g** *x*2 – 3*x* – 40 **h** *x*2 + 3*x* – 28

**3** Factorise

**a** 36*x*2 – 49*y*2 **b** 4*x*2 – 81*y*2

**c** 18*a*2 – 200*b*2*c*2

**4** Factorise

**a** 2*x*2 + *x* –3 **b** 6*x*2 + 17*x* + 5

**c** 2*x*2 + 7*x* + 3 **d** 9*x*2 – 15*x* + 4

**e** 10*x*2 + 21*x* + 9 **f** 12*x*2 – 38*x* + 20

**5** Simplify the algebraic fractions.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**6** Simplify

**a**  **b** 

**c**  **d** 

**Extend**

**7** Simplify 

**8** Simplify 

**Answers**

**1 a** 2*x*3*y*3(3*x* – 5*y*) **b** 7*a*3*b*2(3*b*3 + 5*a*2)

**c** 5*x*2*y*2(5 – 2*x* + 3*y*)

**2** **a** (*x* + 3)(*x* + 4) **b** (*x* + 7)(*x* – 2)

**c** (*x* – 5)(*x* – 6) **d** (*x* – 8)(*x* + 3)

**e** (*x* – 9)(*x* + 2) **f** (*x* + 5)(*x* – 4)

**g** (*x* – 8)(*x* + 5) **h** (*x* + 7)(*x* – 4)

**3 a** (6*x* – 7*y*)(6*x* + 7*y*) **b** (2*x* – 9*y*)(2*x* + 9*y*)

**c** 2(3*a* – 10*bc*)(3*a* + 10*bc*)

**4** **a** (*x* – 1)(2*x* + 3) **b** (3*x* + 1)(2*x* + 5)

**c** (2*x* + 1)(*x* + 3) **d** (3*x* – 1)(3*x* – 4)

**e** (5*x* + 3)(2*x* +3) **f** 2(3*x* – 2)(2*x* –5)

**5 a**  **b** 

**c**  **d** 

**e**  **f** 

**6 a**  **b** 

**c**  **d** 

**7** (*x* + 5)

**8** 

**Completing the square**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square for a quadratic rearranges *ax*2 + *bx* + *c* into the form *p*(*x* + *q*)2 + *r*
* If *a* ≠ 1, then factorise using *a* as a common factor.

Examples

**Example 1** Complete the square for the quadratic expression *x*2 + 6*x* − 2

|  |  |
| --- | --- |
| *x*2 + 6*x* − 2  = (*x* + 3)2 − 9 − 2  = (*x* + 3)2 − 11 | **1** Write *x*2 + *bx* + *c* in the form  **2** Simplify |

**Example 2** Write 2*x*2 − 5*x* + 1 in the form *p*(*x* + *q*)2 + *r*

|  |  |
| --- | --- |
| 2*x*2 − 5*x* + 1  =  =  =  = | **1** Before completing the square write *ax*2 + *bx* + *c* in the form  **2** Now complete the square by writing  in the form  **3** Expand the square brackets – don’t forget to multiply by the factor of 2  **4** Simplify |

Practice

**1** Write the following quadratic expressions in the form (*x* + *p*)2 + *q*

**a** *x*2 + 4*x* + 3 **b** *x*2 – 10*x* – 3

**c** *x*2 – 8*x* **d** *x*2 + 6*x*

**e** *x*2 – 2*x* + 7 **f** *x*2 + 3*x* – 2

**2** Write the following quadratic expressions in the form *p*(*x* + *q*)2 + *r*

**a** 2*x*2 – 8*x* – 16 **b** 4*x*2 – 8*x* – 16

**c** 3*x*2 + 12*x* – 9 **d** 2*x*2 + 6*x* – 8

**3** Complete the square.

**a** 2*x*2 + 3*x* + 6 **b** 3*x*2 – 2*x*

**c** 5*x*2 + 3*x* **d** 3*x*2 + 5*x* + 3

Extend

**4** Write (25*x*2 + 30*x* + 12) in the form (*ax* + *b*)2 + *c*.

Answers

**1 a** (*x* + 2)2 – 1 **b** (*x* – 5)2 – 28

**c** (*x* – 4)2 – 16 **d** (*x* + 3)2 – 9

**e** (*x* – 1)2 + 6 **f** 

**2 a** 2(*x* – 2)2 – 24 **b** 4(*x* – 1)2 – 20

**c** 3(*x* + 2)2 – 21 **d** 

**3 a**  **b** 

**c**  **d** 

**4** (5*x* + 3)2 + 3

**Solving quadratic equations by factorisation**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* A quadratic equation is an equation in the form *ax*2 + *bx* + *c* = 0 where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose products is *ac*.
* When the product of two numbers is 0, then at least one of the numbers must be 0.
* If a quadratic can be solved it will have two solutions (these may be equal).

Examples

**Example 1** Solve 5*x*2 = 15*x*

|  |  |
| --- | --- |
| 5*x*2 = 15*x*  5*x*2 − 15*x* = 0  5*x*(*x* − 3) = 0  So 5*x* = 0 or (*x* − 3) = 0  Therefore *x* = 0 or *x* = 3 | **1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero.  Do not divide both sides by *x* as this would lose the solution *x* = 0.  **2** Factorise the quadratic equation.  5*x* is a common factor.  **3** When two values multiply to make zero, at least one of the values must be zero.  **4** Solve these two equations. |

**Example 2** Solve *x*2 + 7*x* + 12 = 0

|  |  |
| --- | --- |
| *x*2 + 7*x* + 12 = 0  *b* = 7, *ac* = 12  *x*2 + 4*x* + 3*x* + 12 = 0  *x*(*x* + 4) + 3(*x* + 4) = 0  (*x* + 4)(*x* + 3) = 0  So (*x* + 4)= 0 or (*x* + 3) = 0  Therefore *x* = −4 or *x* = −3 | **1** Factorise the quadratic equation. Work out the two factors of *ac* = 12 which add to give you *b* = 7.  (4 and 3)  **2** Rewrite the *b* term (7*x*) using these two factors.  **3** Factorise the first two terms and the last two terms.  **4** (*x* + 4) is a factor of both terms.  **5** When two values multiply to make zero, at least one of the values must be zero.  **6** Solve these two equations. |

**Example 3** Solve 9*x*2 − 16 = 0

|  |  |
| --- | --- |
| 9*x*2 − 16 = 0  (3*x* + 4)(3*x* – 4) = 0  So (3*x* + 4) = 0 or (3*x* – 4) = 0  or | **1** Factorise the quadratic equation. This is the difference of two squares as the two terms are (3*x*)2 and (4)2.  **2** When two values multiply to make zero, at least one of the values must be zero.  **3** Solve these two equations. |

**Example 4** Solve 2*x*2 − 5*x* − 12 = 0

|  |  |
| --- | --- |
| *b* = −5, *ac* = −24  So 2*x*2 − 8*x* + 3*x* – 12 = 0  2*x*(*x* − 4) + 3(*x* − 4) = 0  (*x* – 4)(2*x* + 3) = 0  So (*x* – 4) = 0 or (2*x* +3) = 0  or | **1** Factorise the quadratic equation. Work out the two factors of *ac* = −24 which add to give you *b* = −5.  (−8 and 3)  **2** Rewrite the *b* term (−5*x*) using these two factors.  **3** Factorise the first two terms and the last two terms.  **4** (*x* − 4) is a factor of both terms.  **5** When two values multiply to make zero, at least one of the values must be zero.  **6** Solve these two equations. |

Practice

**1** Solve

**a** 6*x*2 + 4*x* = 0 **b** 28*x*2 – 21*x* = 0

**c** *x*2 + 7*x* + 10 = 0 **d** *x*2 – 5*x* + 6 = 0

**e** *x*2 – 3*x* – 4 = 0 **f** *x*2 + 3*x* – 10 = 0

**g** *x*2 – 10*x* + 24 = 0 **h** *x*2 – 36 = 0

**i** *x*2 + 3*x* – 28 = 0 **j** *x*2 – 6*x* + 9 = 0

**k** 2*x*2 – 7*x* – 4 = 0 **l** 3*x*2 – 13*x* – 10 = 0

**2** Solve

**Hint**

Get all terms onto one side of the equation.

**a** *x*2 – 3*x* = 10 **b** *x*2 – 3 = 2*x*

**c** *x*2 + 5*x* = 24 **d** *x*2 – 42 = *x*

**e** *x*(*x* + 2) = 2*x* + 25 **f** *x*2 – 30 = 3*x* – 2

**g** *x*(3*x* + 1) = *x*2 + 15 **h** 3*x*(*x* – 1) = 2(*x* + 1)

**Solving quadratic equations by completing the square**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square lets you write a quadratic equation in the form *p*(*x* + *q*)2 + *r* = 0*.*

Examples

**Example 5** Solve *x*2 + 6*x* + 4 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| *x*2 + 6*x* + 4 = 0  (*x* + 3)2 − 9 + 4 = 0  (*x* + 3)2 − 5 = 0  (*x* + 3)2 = 5  *x* + 3 =  *x* =  So *x* =  or *x* = | **1** Write *x*2 + *bx* + *c* = 0 in the form  **2** Simplify.  **3** Rearrange the equation to work out *x*. First, add 5 to both sides.  **4** Square root both sides.  Remember that the square root of a value gives two answers.  **5** Subtract 3 from both sides to solve the equation.  **6** Write down both solutions. |

**Example 6** Solve 2*x*2 − 7*x* + 4 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| 2*x*2 − 7*x* + 4 = 0  = 0  = 0  = 0  = 0          So  or | **1** Before completing the square write *ax*2 + *bx* + *c* in the form  **2** Now complete the square by writing  in the form  **3** Expand the square brackets.  **4** Simplify.  *(continued on next page)*  **5** Rearrange the equation to work out *x*. First, add  to both sides.  **6** Divide both sides by 2.  **7** Square root both sides. Remember that the square root of a value gives two answers.  **8** Add  to both sides.  **9** Write down both the solutions. |

Practice

**3** Solve by completing the square.

**a** *x*2 – 4*x* – 3 = 0 **b** *x*2 – 10*x* + 4 = 0

**c** *x*2 + 8*x* – 5 = 0 **d** *x*2 – 2*x* – 6 = 0

**e** 2*x*2 + 8*x* – 5 = 0 **f** 5*x*2 + 3*x* – 4 = 0

**4** Solve by completing the square.

**Hint**

Get all terms onto one side of the equation.

**a** (*x* – 4)(*x* + 2) = 5

**b** 2*x*2 + 6*x* – 7 = 0

**c** *x*2 – 5*x* + 3 = 0

**Solving quadratic equations by using the formula**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Any quadratic equation of the form *ax*2 + *bx* + *c* = 0 can be solved using the formula 
* If *b*2 – 4*ac* is negative then the quadratic equation does not have any real solutions.
* It is useful to write down the formula before substituting the values for *a*, *b* and *c*.

Examples

**Example 7** Solve *x*2 + 6*x* + 4 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| *a* = 1, *b* = 6, *c* = 4            So  or | **1** Identify *a*, *b* and *c* and write down the formula.  Remember that  is all over 2*a*, not just part of it.  **2** Substitute *a* = 1, *b* = 6, *c* = 4 into the formula.  **3** Simplify. The denominator is 2, but this is only because *a* = 1. The denominator will not always be 2.  **4** Simplify .  **5** Simplify by dividing numerator and denominator by 2.  **6** Write down both the solutions. |

**Example 8** Solve 3*x*2 − 7*x* − 2 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| *a* = 3, *b* = −7, *c* = −2        So  or | **1** Identify *a*, *b* and *c*, making sure you get the signs right and write down the formula.  Remember that  is all over 2*a*, not just part of it.  **2** Substitute *a* = 3, *b* = −7, *c* = −2 into the formula.  **3** Simplify. The denominator is 6 when *a* = 3. A common mistake is to always write a denominator of 2.  **4** Write down both the solutions. |

Practice

**5** Solve, giving your solutions in surd form.

**a** 3*x*2 + 6*x* + 2 = 0 **b** 2*x*2 – 4*x* – 7 = 0

**6** Solve the equation *x*2 – 7*x* + 2 = 0

Give your solutions in the form , where *a*, *b* and *c* are integers.

**7** Solve 10*x*2 + 3*x* + 3 = 5

**Hint**

Get all terms onto one side of the equation.

Give your solution in surd form.

Extend

**8** Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

**a** 4*x*(*x* – 1) = 3*x* – 2

**b** 10 = (*x* + 1)2

**c** *x*(3*x* – 1) = 10

Answers

**1 a** *x* = 0 or *x* =  **b** *x* = 0 or *x* = 

**c** *x* = –5 or *x* = –2 **d** *x* = 2 or *x* = 3

**e** *x* = –1 or *x* = 4 **f** *x* = –5 or *x* = 2

**g** *x* = 4 or *x* = 6 **h** *x* = –6 or *x* = 6

**i** *x* = –7 or *x* = 4 **j** *x* = 3

**k** *x* =  or *x* = 4 **l** *x* =  or *x* = 5

**2 a** *x* = –2 or *x* = 5 **b** *x* = –1 or *x* = 3

**c** *x* = –8 or *x* = 3 **d** *x* = –6 or *x* = 7

**e** *x* = –5 or *x* = 5 **f** *x* = –4 or *x* = 7

**g** *x* = –3 or *x* = 2 **h** *x* =  or *x* = 2

**3 a** *x* = 2 + or *x* = 2 –  **b** *x* = 5 +  or *x* = 5 – 

**c** *x* = –4 +  or *x* = –4 –  **d** *x* = 1 +  or *x* = 1 – 

**e** *x* = –2 +  or *x* = –2 –  **f** *x* =  or *x* = 

**4 a** *x* = 1 +  or *x* = 1 –  **b** *x* =  or *x* = 

**c** *x* =  or *x* = 

**5 a** *x* = –1 +  or *x* = –1 –  **b** *x* = 1 +  or *x* = 1 – 

**6** *x* =  or *x* = 

**7** *x* =  or *x* = 

**8 a** *x* =  or *x* = 

**b** *x* = –1 +  or *x* = –1 – 

**c** *x* = –1 or *x* = 2

**Sketching quadratic graphs**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* A black line drawing of a necklace

  AI-generated content may be incorrect.A black line drawing of a necklace

  AI-generated content may be incorrect.The graph of the quadratic function   
  *y* = *ax*2 + *bx* + *c*, where *a* ≠ 0, is a curve   
  called a parabola.

for *a* < 0

for *a* > 0

* Parabolas have a line of symmetry and   
  a shape as shown.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

**Example 1** Sketch the graph of *y* = *x*2.

|  |  |
| --- | --- |
| A graph of a function  AI-generated content may be incorrect. | The graph of *y* = *x*2 is a parabola.  When *x* = 0, *y* = 0.  A black line on a white background  AI-generated content may be incorrect.*a* = 1 which is greater than zero, so the graph has the shape: |

**Example 2** Sketch the graph of *y* = *x*2 − *x* − 6.

|  |  |
| --- | --- |
| When *x* = 0, *y* = 02 − 0 − 6 = −6  So the graph intersects the *y*-axis at  (0, −6)  When *y* = 0, *x*2 − *x* − 6 = 0  (*x* + 2)(*x* − 3) = 0  *x* = −2 or *x* = 3  So,  the graph intersects the *x*-axis at (−2, 0) and (3, 0)  *x*2 − *x* − 6 =  =  When ,  and , so the turning point is at the point  A graph of a parabola  AI-generated content may be incorrect. | **1** Find where the graph intersects the *y*-axis by substituting *x* = 0.  **2** Find where the graph intersects the *x*-axis by substituting *y* = 0.  **3** Solve the equation by factorising.  **4** Solve (*x* + 2) = 0 and (*x* − 3) = 0.  A black line on a white background  AI-generated content may be incorrect.**5** *a* = 1 which is greater than zero, so the graph has the shape:  *(continued on next page)*  **6** To find the turning point, complete the square.  **7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero. |

Practice

**1** Sketch the graph of *y* = −*x*2.

**2** Sketch each graph, labelling where the curve crosses the axes.

**a** *y* = (*x* + 2)(*x* − 1) **b** *y* = *x*(*x* − 3) **c** *y* = (*x* + 1)(*x* + 5)

**3** Sketch each graph, labelling where the curve crosses the axes.

**a** *y* = *x*2 − *x* − 6 **b** *y* = *x*2 − 5*x* + 4 **c** *y* = *x*2 – 4

**d** *y* = *x*2 + 4*x* **e** *y* = 9 − *x*2 **f** *y* = *x*2 + 2*x* − 3

**4** Sketch the graph of *y* = 2*x*2 + 5*x* − 3, labelling where the curve crosses the axes.

Extend

**5** Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

**a** *y* = *x*2 − 5*x* + 6 **b** *y* = −*x*2 + 7*x* − 12 **c** *y* = −*x*2 + 4*x*

**6** Sketch the graph of *y* = *x*2 + 2*x* + 1. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

**1**

A graph of a function

AI-generated content may be incorrect.

**2 a b c**

A graph of a function

AI-generated content may be incorrect. A graph of a function

AI-generated content may be incorrect. A graph of a function

AI-generated content may be incorrect.

**3 a b c**

**A graph of a function

AI-generated content may be incorrect. A graph of a function

AI-generated content may be incorrect. A graph of an x and y axis

AI-generated content may be incorrect.**

**d e f**

**A graph of an x and y axis

AI-generated content may be incorrect.** A graph of a function

AI-generated content may be incorrect. A graph of an equation

AI-generated content may be incorrect.

**4**

A graph of a function

AI-generated content may be incorrect.

**5 a b c**

A graph of a function

AI-generated content may be incorrect. A graph of a function

AI-generated content may be incorrect. A graph of a function

AI-generated content may be incorrect.

**6**

A graph of a function

AI-generated content may be incorrect.

Line of symmetry at *x* = −1.

**Solving linear simultaneous equations using the elimination method**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* Two equations are simultaneous when they are both true at the same time.
* Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
* Make sure that the coefficient of one of the unknowns is the same in both equations.
* Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

**Example 1** Solve the simultaneous equations 3*x* + *y* = 5 and *x* + *y* = 1

|  |  |
| --- | --- |
| 3*x* + *y* = 5  *– x* + *y* = 1  2*x* = 4  So *x* = 2  Using *x* + *y* = 1  2 + *y* = 1  So *y* = −1  Check:  equation 1: 3 × 2 + (−1) = 5 YES  equation 2: 2 + (−1) = 1 YES | **1** Subtract the second equation from the first equation to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 2 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve *x* + 2*y* = 13 and 5*x* − 2*y* = 5 simultaneously.

|  |  |
| --- | --- |
| *x* + 2*y* = 13  + 5*x* − 2*y* = 5     6*x* = 18  So *x* = 3  Using *x* + 2*y* = 13  3 + 2*y* = 13  So *y* = 5  Check:  equation 1: 3 + 2 × 5 = 13 YES  equation 2: 5 × 3 − 2 × 5 = 5 YES | **1** Add the two equations together to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 3 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 3** Solve 2*x* + 3*y* = 2 and 5*x* + 4*y* = 12 simultaneously.

|  |  |
| --- | --- |
| (2*x* + 3*y* = 2) × 4  8*x* + 12*y* = 8  (5*x* + 4*y* = 12) × 3 15*x* + 12*y* = 36  7*x* = 28  So *x* = 4  Using 2*x* + 3*y*  = 2  2 × 4 + 3*y* = 2  So *y* = −2  Check:  equation 1: 2 × 4 + 3 × (−2) = 2 YES  equation 2: 5 × 4 + 4 × (−2) = 12 YES | **1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of *y* the same for both equations. Then subtract the first equation from the second equation to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 4 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** 4*x* + *y* = 8 **2** 3*x* + *y* = 7

*x* + *y* = 5 3*x* + 2*y* = 5

**3** 4*x* + *y* = 3 **4** 3*x* + 4*y* = 7

3*x* – *y* = 11 *x* – 4*y* = 5

**5** 2*x* + *y* = 11 **6** 2*x* + 3*y* = 11

*x* – 3*y* = 9 3*x* + 2*y* = 4

**Solving linear simultaneous equations using the substitution method**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Textbook:**Pure Year 1, 3.1 Linear simultaneous equations

Key points

* The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

**Example 4** Solve the simultaneous equations *y* = 2*x* + 1 and 5*x* + 3*y* = 14

|  |  |
| --- | --- |
| 5*x* + 3(2*x* + 1) = 14  5*x* + 6*x* + 3 = 14  11*x* + 3 = 14  11*x* = 11  So *x* = 1  Using *y* = 2*x* + 1  *y* = 2 × 1 + 1  So *y* = 3  Check:  equation 1: 3 = 2 × 1 + 1 YES  equation 2: 5 × 1 + 3 × 3 = 14 YES | **1** Substitute 2*x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Work out the value of *x*.  **4** To find the value of *y*, substitute *x*= 1 into one of the original equations.  **5** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 5** Solve 2*x* − *y* = 16 and 4*x* + 3*y* = −3 simultaneously.

|  |  |
| --- | --- |
| *y* = 2*x* − 16  4*x* + 3(2*x* − 16) = −3  4*x* + 6*x* − 48 = −3  10*x* − 48 = −3  10*x* = 45  So *x* =  Using *y* = 2*x* − 16  *y* = 2 ×  − 16  So *y* = −7  Check:  equation 1: 2 ×  – (–7) = 16 YES  equation 2: 4 ×  + 3 × (−7) = −3 YES | **1** Rearrange the first equation.  **2** Substitute 2*x* − 16 for *y* into the second equation.  **3** Expand the brackets and simplify.  **4** Work out the value of *x*.  **5** To find the value of *y*, substitute *x*=  into one of the original equations.  **6** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**7** *y* = *x* –4 **8** *y* = 2*x* – 3

2*x* + 5*y* = 43 5*x* – 3*y* = 11

**9** 2*y* = 4*x* + 5 **10** 2*x* = *y* – 2

9*x* + 5*y* = 22 8*x* – 5*y* = –11

**11** 3*x* + 4*y* = 8 **12** 3*y* = 4*x* – 7

2*x* – *y* = –13 2*y* = 3*x* – 4

**13** 3*x* = *y* – 1 **14** 3*x* + 2*y* + 1 = 0

2*y* – 2*x* = 3 4*y* = 8 – *x*

Extend

**15** Solve the simultaneous equations 3*x* + 5*y* − 20 = 0 and .

Answers

**1** *x* = 1, *y* = 4

**2** *x* = 3, *y* = –2

**3** *x* = 2, *y* = –5

**4** *x* = 3, *y* = –

**5** *x* = 6, *y* = –1

**6** *x* = –2, *y* = 5

**7** *x* = 9, *y* = 5

**8** *x* = –2, *y* = –7

**9** *x* = , *y* = 3

**10** *x* = , *y* = 3

**11** *x* = –4, *y* = 5

**12** *x* = –2, *y* = –5

**13** *x* = , *y* = 1

**14** *x* = –2, *y* = 2

**15** *x* = –2, *y* = 5

**Solving linear and quadratic simultaneous equations**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* Make one of the unknowns the subject of the linear equation (rearranging where necessary).
* Use the linear equation to substitute into the quadratic equation.
* There are usually two pairs of solutions.

Examples

**Example 1** Solve the simultaneous equations *y* = *x* + 1 and *x*2 + *y*2 = 13

|  |  |
| --- | --- |
| *x*2 + (*x* + 1)2 = 13  *x*2 + *x*2 + *x* + *x* + 1 = 13  2*x*2 + 2*x* + 1 = 13  2*x*2 + 2*x* − 12 = 0  (2*x* − 4)(*x* + 3) = 0  So *x* = 2 or *x* = −3  Using *y* = *x* + 1  When *x* = 2, *y* = 2 + 1 = 3  When *x* = −3, *y* = −3 + 1 = −2  So the solutions are  *x* = 2, *y* = 3 and *x* = −3, *y* = −2  Check:  equation 1: 3 = 2 + 1 YES  and −2 = −3 + 1 YES  equation 2: 22 + 32 = 13 YES  and (−3)2 + (−2)2 = 13 YES | **1** Substitute *x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Factorise the quadratic equation.  **4** Work out the values of *x*.  **5** To find the value of *y*, substitute both values of *x* into one of the original equations.  **6** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve 2*x* + 3*y* = 5 and 2*y*2 + *xy* = 12 simultaneously.

|  |  |
| --- | --- |
| (*y* + 8)(*y* − 3) = 0  So *y* = −8 or *y* = 3  Using 2*x* + 3*y* = 5  When *y* = −8, 2*x* + 3 × (−8) = 5, *x* = 14.5  When *y* = 3, 2*x* + 3 × 3 = 5, *x* = −2  So the solutions are  *x* = 14.5, *y* = −8 and *x* = −2, *y* = 3  Check:  equation 1: 2 × 14.5 + 3 × (−8) = 5 YES  and 2 × (−2) + 3 × 3 = 5 YES  equation 2: 2×(−8)2 + 14.5×(−8) = 12 YES  and 2 × (3)2 + (−2) × 3 = 12 YES | **1** Rearrange the first equation.  **2** Substitute  for *x* into the second equation. Notice how it is easier to substitute for *x* than for *y*.  **3** Expand the brackets and simplify.  **4** Factorise the quadratic equation.  **5** Work out the values of *y*.  **6** To find the value of *x*, substitute both values of *y* into one of the original equations.  **7** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** *y* = 2*x* + 1 **2** *y* = 6 − *x*

*x*2 + *y*2 = 10 *x*2 + *y*2 = 20

**3** *y* = *x* – 3 **4** *y* = 9 − 2*x*

*x*2 + *y*2 = 5 *x*2 + *y*2 = 17

**5** *y* = 3*x* – 5 **6** *y* = *x* − 5

*y* = *x*2 − 2*x* + 1 *y* = *x*2 − 5*x* − 12

**7** *y* = *x* + 5 **8** *y* = 2*x* – 1

*x*2 + *y*2 = 25 *x*2 + *xy* = 24

**9** *y* = 2*x* **10** 2*x* + *y* = 11

*y*2 – *xy* = 8 *xy* = 15

Extend

**11** *x* – *y* = 1 **12** *y* – *x* = 2

*x*2 + *y*2 = 3 *x*2 + *xy* = 3

Answers

**1** *x* = 1, *y* = 3



**2** *x* = 2, *y* = 4

*x* = 4, *y* = 2

**3** *x* = 1, *y* = −2

*x* = 2, *y* = –1

**4** *x* = 4, *y* = 1



**5** *x* = 3, *y* = 4

*x* = 2, *y* = 1

**6** *x* = 7, *y* = 2

*x* = −1, *y* = −6

**7** *x* = 0, *y* = 5

*x* = –5, *y* = 0

**8** *x* = , *y* = 

*x* = 3, *y* = 5

**9** *x* = –2, *y* = –4

*x* = 2, *y* = 4

**10** *x* = , *y* = 6

*x* = 3, *y* = 5

**11** *x* = , *y* = 

*x* = , *y* = 

**12** *x* = , *y* = 

*x* = , *y* = 

**Solving simultaneous equations graphically**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Key points**

* You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

**Examples**

**Example 1** Solve the simultaneous equations *y* = 5*x* + 2 and *x* + *y* = 5 graphically.

|  |  |
| --- | --- |
| *y* = 5 – *x*  *y* = 5 – *x* has gradient –1 and *y*-intercept 5.  *y* = 5*x* + 2 has gradient 5 and *y*-intercept 2.  A graph of a function  AI-generated content may be incorrect.  Lines intersect at  *x* = 0.5, *y* = 4.5  Check:  First equation *y* = 5*x* + 2:  4.5 = 5 × 0.5 + 2 YES  Second equation *x* + *y* = 5:  0.5 + 4.5 = 5 YES | **1** Rearrange the equation *x* + *y* = 5 to make *y* the subject.  **2** Plot both graphs on the same grid using the gradients and *y*-intercepts.  **3** The solutions of the simultaneous equations are the point of intersection.  **4** Check your solutions by substituting the values into both equations. |

**Example 2** Solve the simultaneous equations *y* = *x* − 4 and *y* = *x*2 − 4*x* + 2 graphically.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | ***x*** | 0 | 1 | 2 | 3 | 4 | | ***y*** | 2 | –1 | –2 | –1 | 2 |   A graph of function with lines and numbers  AI-generated content may be incorrect.  The line and curve intersect at  *x* = 3, *y* = −1 and *x* = 2, *y* = −2  Check:  First equation *y* = *x* − 4:  −1 = 3 − 4 YES  −2 = 2 − 4 YES  Second equation *y* = *x*2 − 4*x* + 2:  −1 = 32 − 4 × 3 + 2 YES  −2 = 22 − 4 × 2 + 2 YES | **1** Construct a table of values and calculate the points for the quadratic equation.  **2** Plot the graph.  **3** Plot the linear graph on the same grid using the gradient and  *y*-intercept. *y* = *x* – 4 has gradient 1 and  *y*-intercept –4.  **4** The solutions of the simultaneous equations are the points of intersection.  **5** Check your solutions by substituting the values into both equations. |

**Practice**

**1** Solve these pairs of simultaneous equations graphically.

**a** *y* = 3*x* − 1 and *y* = *x* + 3

**b** *y* = *x* − 5 and *y* = 7 − 5*x*

**c** *y* = 3*x* + 4 and *y* = 2 − *x*

**2** Solve these pairs of simultaneous equations graphically.

**Hint**

Rearrange the equation to make *y* the subject.

**a** *x* + *y* = 0 and *y* = 2*x* + 6

**b** 4*x* + 2*y* = 3 and *y* = 3*x* − 1

**c** 2*x* + *y* + 4 = 0 and 2*y* = 3*x* − 1

**3** Solve these pairs of simultaneous equations graphically.

**a** *y* = *x* − 1 and *y* = *x*2 − 4*x* + 3

**b** *y* = 1 − 3*x* and *y* = *x*2 − 3*x* − 3

**c** *y* = 3 − *x* and *y* = *x*2 + 2*x* + 5

**4** Solve the simultaneous equations *x* + *y* = 1 and *x*2 + *y*2 = 25 graphically.

**Extend**

**5** **a** Solve the simultaneous equations 2*x* + *y* = 3 and *x*2 + *y* = 4

**i** graphically

**ii** algebraically to 2 decimal places.

**b** Which method gives the more accurate solutions? Explain your answer.

**Answers**

**1 a** *x* = 2, *y* = 5

**b** *x* = 2, *y* = −3

**c** *x* = −0.5, *y* = 2.5

**2 a** *x* = −2, *y* = 2

**b** *x* = 0.5, *y* = 0.5

**c** *x* = −1, *y* = −2

**3 a** *x* = 1, *y* = 0 and *x* = 4, *y* = 3

**b** *x* = −2, *y* = 7 and *x* = 2, *y* = −5

**c** *x* = −2, *y* = 5 and *x* = −1, *y* = 4

**4** *x* = −3, *y* = 4 and *x* = 4, *y* = −3

**5 a i** *x* = 2.5, *y* = −2 and *x* = −0.5, *y* = 4

**ii** *x* = 2.41, *y* = −1.83 and *x* = −0.41, *y* = 3.83

**b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

**Linear inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

**Key points**

* Solving linear inequalities uses similar methods to those for solving linear equations.
* When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

**Examples**

**Example 1** Solve −8 ≤ 4*x* < 16

|  |  |
| --- | --- |
| −8 ≤ 4*x* < 16  −2 ≤ *x*  < 4 | Divide all three terms by 4. |

**Example 2** Solve 4 ≤ 5*x* < 10

|  |  |
| --- | --- |
| 4 ≤ 5*x* < 10  ≤ *x* < 2 | Divide all three terms by 5. |

**Example 3** Solve 2*x* − 5 < 7

|  |  |
| --- | --- |
| 2*x* − 5 < 7  2*x* < 12  *x* < 6 | **1** Add 5 to both sides.  **2** Divide both sides by 2. |

**Example 4** Solve 2 − 5*x* ≥ −8

|  |  |
| --- | --- |
| 2 − 5*x* ≥ −8  −5*x* ≥ −10  *x* ≤ 2 | **1** Subtract 2 from both sides.  **2** Divide both sides by −5.  Remember to reverse the inequality when dividing by a negative number. |

**Example 5** Solve 4(*x* − 2) > 3(9 − *x*)

|  |  |
| --- | --- |
| 4(*x* − 2) > 3(9 − *x*)  4*x* − 8 > 27 − 3*x*  7*x* − 8 > 27  7*x* > 35  *x* > 5 | **1** Expand the brackets.  **2** Add 3*x* to both sides.  **3** Add 8 to both sides.  **4** Divide both sides by 7. |

**Practice**

**1** Solve these inequalities.

**a** 4*x* > 16 **b** 5*x* – 7 ≤ 3 **c** 1 ≥ 3*x* + 4

**d** 5 – 2*x* < 12 **e**  **f** 8 < 3 – 

**2** Solve these inequalities.

**a**  **b** 10 ≥ 2*x* + 3 **c** 7 – 3*x* > –5

**3** Solve

**a** 2 – 4*x* ≥ 18 **b** 3 ≤ 7*x* + 10 < 45 **c** 6 – 2*x* ≥ 4

**d** 4*x* + 17 < 2 – *x* **e** 4 – 5*x* < –3*x* **f** –4*x* ≥ 24

**4** Solve these inequalities.

**a** 3*t* + 1 < *t* + 6 **b** 2(3*n* – 1) ≥ *n* + 5

**5** Solve.

**a** 3(2 – *x*) > 2(4 – *x*) + 4 **b** 5(4 – *x*) > 3(5 – *x*) + 2

**Extend**

**6** Find the set of values of *x* for which 2*x* + 1 > 11 and 4*x* – 2 > 16 – 2*x*.

**Answers**

**1** **a** *x* > 4 **b** *x* ≤ 2 **c** *x* ≤ –1

**d** *x* > – **e** *x* ≥ 10 **f** *x* < –15

**2 a** *x* < –20 **b** *x* ≤ 3.5 **c** *x* < 4

**3 a** *x* ≤ –4 **b** –1 ≤ *x* < 5 **c** *x* ≤ 1

**d** *x* < –3 **e** *x* > 2 **f** *x* ≤ –6

**4 a** *t* <  **b** *n* ≥ 

**5 a** *x* < –6 **b** *x* < 

**6** *x* > 5 (which also satisfies *x* > 3)

**Quadratic inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

**Key points**

* First replace the inequality sign by = and solve the quadratic equation.
* Sketch the graph of the quadratic function.
* Use the graph to find the values which satisfy the quadratic inequality.

**Examples**

**Example 1** Find the set of values of *x* which satisfy *x*2 + 5*x* + 6 > 0

|  |  |
| --- | --- |
| *x*2 + 5*x* + 6 = 0  (*x* + 3)(*x* + 2) = 0  *x* = −3 or *x* = −2  A graph of a function  AI-generated content may be incorrect.  *x* < −3 or *x* > −2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of  *y* = (*x* + 3)(*x* + 2)  **3** Identify on the graph where  *x*2 + 5*x* + 6 > 0, i.e. where *y* > 0  **4** Write down the values which satisfy the inequality *x*2 + 5*x* + 6 > 0 |

**Example 2** Find the set of values of *x* which satisfy *x*2 − 5*x* ≤ 0

|  |  |
| --- | --- |
| *x*2 − 5*x* = 0  *x*(*x* − 5) = 0  *x* = 0 or *x* = 5  A graph of a function  AI-generated content may be incorrect.  0 ≤ *x* ≤ 5 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = *x*(*x* − 5)  **3** Identify on the graph where  *x*2 − 5*x* ≤ 0, i.e. where *y* ≤ 0  **4** Write down the values which satisfy the inequality *x*2 − 5*x* ≤ 0 |

**Example 3** Find the set of values of *x* which satisfy −*x*2 − 3*x* + 10 ≥ 0

|  |  |
| --- | --- |
| −*x*2 − 3*x* + 10 = 0  (−*x* + 2)(*x* + 5) = 0  *x* = 2 or *x* = −5    −5 ≤ *x* ≤ 2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = (−*x* + 2)(*x* + 5) = 0  **3** Identify on the graph where −*x*2 − 3*x* + 10 ≥ 0, i.e. where *y* ≥ 0  **3** Write down the values which satisfy the inequality −*x*2 − 3*x* + 10 ≥ 0 |

**Practice**

**1** Find the set of values of *x* for which (*x* + 7)(*x* – 4) ≤ 0

**2** Find the set of values of *x* for which *x*2 – 4*x* – 12 ≥ 0

**3** Find the set of values of *x* for which 2*x*2 –7*x* + 3 < 0

**4** Find the set of values of *x* for which 4*x*2 + 4*x* – 3 > 0

**5** Find the set of values of *x* for which 12 + *x* – *x*2 ≥ 0

**Extend**

Find the set of values which satisfy the following inequalities.

**6** *x*2 + *x* ≤ 6

**7** *x*(2*x* – 9) < –10

**8** 6*x*2 ≥ 15 + *x*

**Answers**

**1** –7≤ *x* ≤ 4

**2** *x* ≤ –2 or *x* ≥ 6

**3 **

**4** *x* <  or *x* > 

**5** –3 ≤ *x* ≤ 4

**6** –3 ≤ *x* ≤ 2

**7** 2 < *x* < 2

**8**  or 

**Sketching cubic and reciprocal graphs**

**A LEVEL LINKS**

**Scheme of work:** 1e. Graphs – cubic, quartic and reciprocal

A group of math equations

AI-generated content may be incorrect.Key points

* The graph of a cubic function, which can be written in the form *y* = *ax*3 + *bx*2 + *cx* + *d*, where *a* ≠ 0, has one of the shapes shown here.

A group of math equations

AI-generated content may be incorrect.

* The graph of a reciprocal function of the form  has one of the shapes shown here.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* Where appropriate, mark and label the asymptotes on the graph.
* Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of  are the two axes (the lines *y* = 0 and *x* = 0).
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* A double root is when two of the solutions are equal. For example (*x* – 3)2(*x* + 2) has a double root at *x* = 3.
* When there is a double root, this is one of the turning points of a cubic function.

Examples

**Example 1** Sketch the graph of *y* = (*x* − 3)(*x* − 1)(*x* + 2)

|  |  |
| --- | --- |
| To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape. | |
| When *x* = 0, *y* = (0 − 3)(0 − 1)(0 + 2)  = (−3) × (−1) × 2 = 6  The graph intersects the *y*-axis at (0, 6)  When *y* = 0, (*x* − 3)(*x* − 1)(*x* + 2) = 0  So *x* = 3, *x* = 1 or *x* = −2  The graph intersects the *x*-axis at   (−2, 0), (1, 0) and (3, 0)  A graph of a function  AI-generated content may be incorrect. | **1** Find where the graph intersects the axes by substituting *x* = 0 and *y* = 0.  Make sure you get the coordinates the right way around, (*x*, *y*).  **2** Solve the equation by solving  *x* − 3 = 0, *x* − 1 = 0 and *x* + 2 = 0  **3** Sketch the graph.  *a* = 1 > 0 so the graph has the shape:  A graph of a function  AI-generated content may be incorrect. |

**Example 2** Sketch the graph of *y* = (*x* + 2)2(*x* − 1)

|  |  |
| --- | --- |
| To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape. | |
| When *x* = 0, *y* = (0 + 2)2(0 − 1)  = 22 × (−1) = −4  The graph intersects the *y*-axis at (0, −4)  When *y* = 0, (*x* + 2)2(*x* − 1) = 0  So *x* = −2 or *x* =1  (−2, 0) is a turning point as *x* = −2 is a double root. The graph crosses the *x*-axis at (1, 0)  A graph of a function  AI-generated content may be incorrect. | **1** Find where the graph intersects the axes by substituting *x* = 0 and *y* = 0.  **2** Solve the equation by solving  *x* + 2 = 0 and *x* − 1 = 0  **3** *a* = 1 > 0 so the graph has the shape:  A graph of a function  AI-generated content may be incorrect. |

Practice

**1** Here are six equations.

**Hint**

Find where each of the cubic equations cross the *y*-axis.

**A**  **B** *y* = *x*2 + 3*x* – 10 **C** *y* = *x*3 + 3*x*2

**D** *y* = 1 – 3*x*2 – *x*3 **E** *y* = *x*3 – 3*x*2 – 1 **F** *x* + *y* = 5

A graph of a function

AI-generated content may be incorrect. Here are six graphs.

**A diagram of a function

AI-generated content may be incorrect.A graph of normal and normal

AI-generated content may be incorrect. i ii iii**

**A graph of a function

AI-generated content may be incorrect.**

**A graph of a function

AI-generated content may be incorrect.A graph of a function

AI-generated content may be incorrect. iv v vi**

**a** Match each graph to its equation.

**b** Copy the graphs ii, iv and vi and draw the tangent and normal each at point *P*.

Sketch the following graphs

**2**  *y* = 2*x*3 **3** *y* = *x*(*x* – 2)(*x* + 2)

**4** *y* = (*x* + 1)(*x* + 4)(*x* – 3) **5** *y* = (*x* + 1)(*x* – 2)(1 – *x*)

**6** *y* = (*x* – 3)2(*x* + 1) **7** *y* = (*x* – 1)2(*x* – 2)

**8** *y* =  **9** *y* = 

**Hint:** Look at the shape of *y* =  in the second key point.

Extend

**10** Sketch the graph of  **11** Sketch the graph of 

Answers

**1****a** i – C

ii – E

iii – B

iv – A

v – F

vi – D

**A diagram of a function

AI-generated content may be incorrect.A graph of a function

AI-generated content may be incorrect. b ii iv**

**A diagram of a function

AI-generated content may be incorrect.**

**vi**

**A graph of a function

AI-generated content may be incorrect.A graph of a function

AI-generated content may be incorrect.2 3**

**A graph of a function

AI-generated content may be incorrect.**

**A graph of a function

AI-generated content may be incorrect.4 5**

**A graph of a function

AI-generated content may be incorrect.A graph of a function

AI-generated content may be incorrect.6 7**

**A graph of a function

AI-generated content may be incorrect.8 9**

**A graph of a function

AI-generated content may be incorrect.A graph of function and numbers

AI-generated content may be incorrect.10 11**

**Translating graphs**

**A LEVEL LINKS**

**Scheme of work:** 1f. Transformations – transforming graphs – f(*x*) notation

A graph of function and equations

AI-generated content may be incorrect.Key points

* The transformation *y* = f(*x*) ± *a* is a translation of *y* = f(*x*) parallel to the *y*-axis; it is a vertical translation.

As shown on the graph,

* + *y* = f(*x*) + *a* translates *y* = f(*x*) up
  + *y* = f(*x*) – *a* translates *y* = f(*x*) down.
* A graph of function and the function of a function

  AI-generated content may be incorrect.The transformation *y =* f(*x ± a*) is a translation of *y* = f(*x*) parallel to the *x*-axis; it is a horizontal translation.

As shown on the graph,

* + *y* = f(*x* + *a*) translates *y* = f(*x*) to the left
  + *y* = f(*x* – *a*) translates *y* = f(*x*) to the right.

A graph of a function

AI-generated content may be incorrect.Examples

**Example 1** The graph shows the function *y* = f(*x*).

Sketch the graph of *y* = f(*x*) + 2.

|  |  |
| --- | --- |
| A graph of a function  AI-generated content may be incorrect. | For the function *y* = f(*x*) + 2 translate the function *y* = f(*x*) 2 units up. |

A graph of function and function

AI-generated content may be incorrect.**Example 2** The graph shows the function *y* = f(*x*).

Sketch the graph of *y* = f(*x* − 3).

|  |  |
| --- | --- |
| A graph of function and equations  AI-generated content may be incorrect. | For the function *y* = f(*x* − 3) translate the function *y* = f(*x*) 3 units right. |

A graph of a function

AI-generated content may be incorrect.Practice

**1** The graph shows the function *y* = f(*x*).   
Copy the graph and on the same axes sketch and label the graphs of *y* = f(*x*) + 4 and *y* = f(*x* + 2).

A graph of a function

AI-generated content may be incorrect.

**2** The graph shows the function *y* = f(*x*).  
Copy the graph and on the same axes sketch and label the graphs of *y* = f(*x* + 3) and *y* = f(*x*) – 3.

A graph of function with numbers and equations

AI-generated content may be incorrect.

**3** The graph shows the function *y* = f(*x*).  
Copy the graph and on the same axes sketch the graph of *y* = f(*x* – 5).

A graph of function and function

AI-generated content may be incorrect.**4** The graph shows the function *y* = f(*x*) and two transformations of *y* = f(*x*), labelled *C*1 and *C*2.  
Write down the equations of the translated curves *C*1 and *C*2 in function form.

A graph of a function

AI-generated content may be incorrect.

**5** The graph shows the function *y* = f(*x*) and two transformations of *y* = f(*x*), labelled *C*1 and *C*2.  
Write down the equations of the translated curves *C*1 and *C*2 in function form.

A graph of function in a grid

AI-generated content may be incorrect.

**6** The graph shows the function *y* = f(*x*).

**a** Sketch the graph of *y* = f(*x*) + 2

**b** Sketch the graph of *y* = f(*x* + 2)

**Stretching graphs**

**A LEVEL LINKS**

**Scheme of work:** 1f. Transformations – transforming graphs – f(*x*) notation

**Textbook:**Pure Year 1, 4.6 Stretching graphs

A graph of mathematical equations

AI-generated content may be incorrect.Key points

* The transformation *y* = f(*ax*) is a horizontal stretch of *y* = f(*x*) with scale factor  parallel to the *x*-axis.

A graph of mathematical equations

AI-generated content may be incorrect.

* The transformation *y* = f(–*ax*) is a horizontal stretch of *y* = f(*x*) with scale factor  parallel to the *x*-axis and then a reflection in the *y*-axis.

A graph of function and equations

AI-generated content may be incorrect.

* The transformation *y* = *a*f(*x*) is a vertical stretch of *y* = f(*x*) with scale factor *a* parallel to the *y*-axis.

A graph of function and mathematical equations

AI-generated content may be incorrect.

* The transformation *y* = –*a*f(*x*) is a vertical stretch of *y* = f(*x*) with scale factor *a* parallel to the *y*-axis and then a reflection in the *x*-axis.

A graph of a function

AI-generated content may be incorrect.Examples

**Example 3** The graph shows the function *y* = f(*x*).

Sketch and label the graphs of   
 *y* = 2f(*x*) and *y* = –f(*x*).

|  |  |
| --- | --- |
| A graph of a function  AI-generated content may be incorrect. | The function *y* = 2f(*x*) is a vertical stretch of *y* = f(*x*) with scale factor 2 parallel to the *y*-axis.  The function *y* = −f(*x*) is a reflection of *y* = f(*x*) in the  *x*-axis. |

A graph of a function

AI-generated content may be incorrect.**Example 4** The graph shows the function *y* = f(*x*).

Sketch and label the graphs of   
 *y* = f(2*x*) and *y* = f(–*x*).

|  |  |
| --- | --- |
| A graph of function in a grid  AI-generated content may be incorrect. | The function *y* = f(2*x*) is a horizontal stretch of *y* = f(*x*) with scale factor  parallel to the *x*-axis.  The function *y* = f(−*x*) is a reflection of *y* = f(*x*) in the *y*-axis. |

A graph of a mathematical equation

AI-generated content may be incorrect.Practice

**7** The graph shows the function *y* = f(*x*).

**a** Copy the graph and on the same axes sketch and label the graph of *y* = 3f(*x*).

**b** Make another copy of the graph and on the same axes sketch and label the graph of *y* = f(2*x*).

A graph of a function

AI-generated content may be incorrect.

**8** The graph shows the function *y* = f(*x*).  
Copy the graph and on the same axes   
sketch and label the graphs of  
*y* = –2f(*x*) and *y* = f(3*x*).

A graph of a function

AI-generated content may be incorrect.

**9** The graph shows the function *y* = f(*x*).   
Copy the graph and, on the same axes,   
sketch and label the graphs of   
*y* = –f(*x*) and *y* =.

A graph of a function

AI-generated content may be incorrect.**10** The graph shows the function *y* = f(*x*).  
Copy the graph and, on the same axes,   
sketch the graph of *y* = –f(2*x*).

A graph of a function

AI-generated content may be incorrect.

**11** The graph shows the function *y* = f(*x*) and a transformation, labelled *C*.  
Write down the equation of the translated curve *C* in function form.

A graph of function and equations

AI-generated content may be incorrect.**12** The graph shows the function *y* = f(*x*) and a transformation labelled *C*.  
Write down the equation of the translated curve *C* in function form.

A graph of a function

AI-generated content may be incorrect.

**13** The graph shows the function *y* = f(*x*).

**a** Sketch the graph of *y* = −f(*x*).

**b** Sketch the graph of *y* = 2f(*x*).

Extend

**14 a** Sketch and label the graph of *y* = f(*x*), where f(*x*) = (*x* – 1)(*x* + 1).

**b** On the same axes, sketch and label the graphs of *y* = f(*x*) – 2 and *y* = f(*x* + 2).

**15** **a** Sketch and label the graph of *y* = f(*x*), where f(*x*) = –(*x* + 1)(*x* – 2).

**b** On the same axes, sketch and label the graph of *y* = .

Answers

A graph of function and equations

AI-generated content may be incorrect.**1****2**

A graph of equations and equations

AI-generated content may be incorrect.

**3**

A graph of function with numbers and lines

AI-generated content may be incorrect.

**4** *C*1: *y* = f(*x* – 90°)  
 *C*2: *y* = f(*x*) – 2

**5** *C*1: *y* = f(*x* – 5)  
 *C*2: *y* = f(*x*) – 3

**6 a b**

A graph of function in a grid

AI-generated content may be incorrect. A graph of function in a grid

AI-generated content may be incorrect.

**7 a b**

A graph of an oval with a circle and a circle with a point in the center

AI-generated content may be incorrect. A graph of a mathematical equation

AI-generated content may be incorrect.

**8 9**

A graph of a function

AI-generated content may be incorrect. A graph of function in a grid

AI-generated content may be incorrect.

**10**

**A graph of a function

AI-generated content may be incorrect.**

**11** *y* = f(2*x*)

**12** *y* = –2f(2*x*) or *y* = 2f(–2*x*)

**13 a b**

A graph of a function

AI-generated content may be incorrect. A graph of function in a grid

AI-generated content may be incorrect.

**14**

A diagram of a function

AI-generated content may be incorrect.

**15**

A graph of function in a grid

AI-generated content may be incorrect.

**Straight line graphs**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

A math equations on a white background

AI-generated content may be incorrect.Key points

* A straight line has the equation *y* = *mx* + *c*, where *m* is the gradient and *c* is the *y*-intercept (where *x* = 0).
* The equation of a straight line can be written in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.
* When given the coordinates (*x*1, *y*1) and (*x*2, *y*2) of two points on a line the gradient is calculated using the formula 

Examples

**Example 1** A straight line has gradient  and *y*-intercept 3.  
Write the equation of the line in the form *ax* + *by* + *c* = 0.

|  |  |
| --- | --- |
| *m* =  and *c* = 3  So *y* = *x* + 3  *x* + *y* – 3 = 0  *x* + 2*y* − 6 = 0 | **1** A straight line has equation *y*= *mx*+ *c*. Substitute the gradient and *y*-intercept given in the question into thisequation.  **2** Rearrange the equation so all the terms are on one side and 0 is on  the other side.  **3** Multiply both sides by 2 to eliminate the denominator. |

**Example 2** Find the gradient and the *y*-intercept of the line with the equation 3*y* − 2*x* + 4 = 0.

|  |  |
| --- | --- |
| 3*y* − 2*x* + 4 = 0  3*y* = 2*x* − 4    Gradient = *m* =  *y*-intercept = *c* = | **1** Make *y* the subject of the equation.  **2** Divide all the terms by three to get the equation in the form *y* = …  **3** In the form *y* = *mx* + *c*, the gradient is *m* and the *y*-intercept is *c*. |

**Example 3** Find the equation of the line which passes through the point (5, 13) and has gradient 3.

|  |  |
| --- | --- |
| *m* = 3  *y* = 3*x* + *c*  13 = 3 × 5 + *c*  13 = 15 + *c*  *c* = −2  *y* = 3*x* − 2 | **1** Substitute the gradient given in the question into the equation of a straight line *y* = *mx* + *c*.  **2** Substitute the coordinates *x* = 5 and *y* = 13 into the equation.  **3** Simplify and solve the equation.  **4** Substitute *c* = −2 into the equation *y*= 3*x*+ *c* |

**Example 4** Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

|  |  |
| --- | --- |
| , ,  and        *c* = 3 | **1** Substitute the coordinates into the equation  to work out the gradient of the line.  **2** Substitute the gradient into the equation of a straight line *y*= *mx*+ *c*.  **3** Substitute the coordinates of either point into the equation.  **4** Simplify and solve the equation.  **5** Substitute *c* = 3 into the equation |

Practice

**1** Find the gradient and the *y*-intercept of the following equations.

**a** *y* = 3*x* + 5 **b** *y* = *x* – 7

**Hint**

Rearrange the equations to the form *y* = *mx* + *c*

**c** 2*y* = 4*x* – 3 **d** *x* + *y* = 5

**e** 2*x* – 3*y* – 7 = 0 **f** 5*x* + *y* – 4 = 0

**2** Copy and complete the table, giving the equation of the line in the form *y* = *mx* + *c*.

|  |  |  |
| --- | --- | --- |
| **Gradient** | ***y*-intercept** | **Equation of the line** |
| 5 | 0 |  |
| –3 | 2 |  |
| 4 | –7 |  |

**3** Find, in the form *ax* + *by* + *c* = 0 where *a*, *b* and *c* are integers, an equation for each of the lines with the following gradients and *y*-intercepts.

**a** gradient , *y*-intercept –7 **b** gradient 2, *y*-intercept 0

**c** gradient , *y*-intercept 4 **d** gradient –1.2, *y*-intercept –2

**4** Write an equation for the line which passes though the point (2, 5) and has gradient 4.

**5** Write an equation for the line which passes through the point (6, 3) and has gradient 

**6** Write an equation for the line passing through each of the following pairs of points.

**a** (4, 5), (10, 17) **b** (0, 6), (–4, 8)

**c** (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

**7** The equation of a line is 2*y* + 3*x* – 6 = 0.  
Write as much information as possible about this line.

Answers

**1** **a** *m* = 3, *c* = 5 **b** *m* = , *c* = –7

**c** *m* = 2, *c* =  **d** *m* = –1, *c* = 5

**e** *m* = , *c* = or –2 **f** *m* = –5, *c* = 4

**2**

|  |  |  |
| --- | --- | --- |
| **Gradient** | ***y*-intercept** | **Equation of the line** |
| 5 | 0 | *y* = 5*x* |
| –3 | 2 | *y* = –3*x* + 2 |
| 4 | –7 | *y* = 4*x* –7 |

**3 a** *x* + 2*y* + 14 = 0 **b** 2*x* – *y* = 0

**c** 2*x* – 3*y* + 12 = 0 **d** 6*x* + 5*y* + 10 = 0

**4** *y* = 4*x* – 3

**5** *y* = *x* + 7

**6 a** *y* = 2*x* – 3 **b** *y* = *x* + 6

**c** *y* = 5*x* –2 **d** *y* = –3*x* + 19

**7** , the gradient is  and the *y*-intercept is 3.  
The line intercepts the axes at (0, 3) and (2, 0).  
Students may sketch the line or give coordinates that lie on the line such as  or .

**Parallel and perpendicular lines**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

**A diagram of a graph

AI-generated content may be incorrect.Key points**

* When lines are parallel they have the same gradient.
* A line perpendicular to the line with equation *y* = *mx* + *c* has gradient .

**Examples**

**Example 1** Find the equation of the line parallel to *y* = 2*x* + 4 which passes through   
the point (4, 9).

|  |  |
| --- | --- |
| *y* = 2*x* + 4  *m* = 2  *y* = 2*x* + *c*  9 = 2 × 4 + *c*  9 = 8 + *c*  *c* = 1  *y* = 2*x* + 1 | **1** As the lines are parallel they have the same gradient.  **2** Substitute *m* = 2 into the equation of a straight line *y* = *mx* + *c*.  **3** Substitute the coordinates into the equation *y* = 2*x* + *c*  **4** Simplify and solve the equation.  **5** Substitute *c* = 1 into the equation *y*= 2*x* + *c* |

**Example 2** Find the equation of the line perpendicular to *y* = 2*x* − 3 which passes through   
the point (−2, 5).

|  |  |
| --- | --- |
| *y* = 2*x* − 3  *m* = 2        5 = 1 + *c*  *c* = 4 | **1** As the lines are perpendicular, the gradient of the perpendicular line  is .  **2** Substitute *m* =  into *y* = *mx* + *c*.  **3** Substitute the coordinates (–2, 5) into the equation  **4** Simplify and solve the equation.  **5** Substitute *c* = 4 into . |

**Example 3** A line passes through the points (0, 5) and (9, −1).  
Find the equation of the line which is perpendicular to the line and passes through   
its midpoint.

|  |  |
| --- | --- |
| , ,  and        Midpoint = | **1** Substitute the coordinates into the equation  to work out the gradient of the line.  **2** As the lines are perpendicular, the gradient of the perpendicular line  is .  **3** Substitute the gradient into the equation *y* = *mx* + *c*.  **4** Work out the coordinates of the midpoint of the line.  **5** Substitute the coordinates of the midpoint into the equation.  **6** Simplify and solve the equation.  **7** Substitute  into the equation . |

**Practice**

**1** Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

**a** *y* = 3*x* + 1 (3, 2) **b** *y* = 3 – 2*x* (1, 3)

**c** 2*x* + 4*y* + 3 = 0 (6, –3) **d** 2*y* –3*x* + 2 = 0 (8, 20)

**2** Find the equation of the line perpendicular to *y* = *x* – 3 which passes through the point (–5, 3).

**Hint**

If *m* =  then the negative reciprocal 

**3** Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

**a** *y* = 2*x* – 6 (4, 0) **b** *y* = *x* +  (2, 13)

**c** *x* –4*y* – 4 = 0 (5, 15) **d** 5*y* + 2*x* – 5 = 0 (6, 7)

**4** In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

**a** (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

**Extend**

**5** Work out whether these pairs of lines are parallel, perpendicular or neither.

**a** *y* = 2*x* + 3 **b** *y* = 3*x* **c** *y* = 4*x* – 3  
 *y* = 2*x* – 7 2*x + y* – 3 = 0 4*y* + *x* = 2

**d** 3*x* – *y* + 5 = 0 **e** 2*x* + 5*y* – 1 = 0 **f** 2*x* – *y* = 6

*x* + 3*y* = 1 *y* = 2*x* + 7 6*x* – 3*y* + 3 = 0

**6** The straight line **L1** passes through the points *A* and *B* with coordinates (–4, 4) and (2, 1), respectively.

**a** Find the equation of **L1** in the form *ax* + *by* + *c* = 0

The line **L2** is parallel to the line **L1** and passes through the point *C* with coordinates (–8, 3).

**b** Find the equation of **L2** in the form *ax* + *by* + *c* = 0

The line **L3** is perpendicular to the line **L1** and passes through the origin.

**c** Find an equation of **L3**

**Answers**

**1 a** *y* = 3*x* –7 **b** *y* = –2*x* + 5

**c** *y* = –*x* **d** *y* = *x* + 8

**2** *y* = −2*x* – 7

**3** **a** *y* = –*x* + 2 **b** *y* = 3*x* + 7

**c** *y* = –4*x* + 35 **d** *y* = *x* – 8

**4** **a** *y* = –*x* **b** *y* = 2*x*

**5** **a** Parallel **b** Neither **c** Perpendicular

**d** Perpendicular **e** Neither **f** Parallel

**6** **a** *x* + 2*y* – 4 = 0 **b** *x* + 2*y* + 2 = 0 **c** *y* = 2*x*

**Pythagoras’ theorem**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

**Key points**

A black triangle with letters and numbers

AI-generated content may be incorrect.In a right-angled triangle the longest side is called the hypotenuse.

Pythagoras’ theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.  
*c*2 = *a*2 + *b*2

**A black triangle with a square and a square

AI-generated content may be incorrect.Examples**

**Example 1** Calculate the length of the hypotenuse.  
Give your answer to 3 significant figures.

|  |  |
| --- | --- |
| *c*2 = *a*2 + *b*2  A black triangle with text and numbers  AI-generated content may be incorrect.  *x*2 = 52 + 82  *x*2 = 25 + 64  *x*2 = 89    *x* = 9.433 981 13...  *x* = 9.43 cm | **1** Always start by stating the formula for Pythagoras’ theorem and labelling the hypotenuse *c* and the other two sides *a* and *b*.  **2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem.  **3** Use a calculator to find the square root.  **4** Round your answer to 3 significant figures and write the units with your answer. |

A black triangle with black text

AI-generated content may be incorrect.**Example 2** Calculate the length *x*.   
Give your answer in surd form.

|  |  |
| --- | --- |
| *c*2 = *a*2 + *b*2  102 = *x*2 + 42  100 = *x*2 + 16  *x*2 = 84    cm | **1** Always start by stating the formula for Pythagoras' theorem.  **2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem.  **3** Simplify the surd where possible and write the units in your answer. |

**Practice**

**1** Work out the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A triangle with text on it

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect. **a b b**

**c d**

A black and white triangle with black text

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect.

**2** Work out the length of the unknown side in each triangle.  
 Give your answers in surd form.

**a b**

A triangle with a number of letters and numbers

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect. **a b**

**c d**

A black line with numbers and a rectangular object

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect.

**3** Work out the length of the unknown side in each triangle.   
 Give your answers in surd form.

A black and white image of a triangle

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect. **a b b**

A black triangle with black text

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect. **c d**

**4** A rectangle has length 84 mm and width 45 mm.   
 Calculate the length of the diagonal of the rectangle.  
 Give your answer correct to 3 significant figures.

**Hint**

Draw a sketch of the rectangle.

**Extend**

**5** A yacht is 40 km due North of a lighthouse.  
A rescue boat is 50 km due East of the same lighthouse.  
Work out the distance between the yacht and the rescue boat.  
Give your answer correct to 3 significant figures.

**Hint**

Draw a diagram using the information given in the question.

**6** Points A and B are shown on the diagram.  
Work out the length of the line AB.   
Give your answer in surd form.

A graph of a function

AI-generated content may be incorrect.

A black and white drawing of a cube with lines and letters

AI-generated content may be incorrect.**7** A cube has length 4 cm.   
Work out the length of the diagonal *AG*.  
Give your answer in surd form.

**Answers**

**1 a** 10.3 cm **b** 7.07 cm

**c** 58.6 mm **d** 8.94 cm

**2 a**  cm **b**  cm

**c**  mm **d**  mm

**3 a**  mm **b**  mm

**c**  mm **d**  mm

**4** 95.3 mm

**5** 64.0 km

**6**  units

**7**  cm

**Proportion**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

**A graph of a graph of a graph

AI-generated content may be incorrect.Key points**

Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate.  
Their ratio remains the same.

‘*y* is directly proportional to *x*’ is written as *y*  *x*.  
If *y*  *x* then *y* = *kx*, where *k* is a constant.

When *x* is directly proportional to *y*, the graph is a straight line passing through the origin.

A graph of a function

AI-generated content may be incorrect.

Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.

‘*y* is inversely proportional to *x*’ is written as *y*  .   
If *y*   then *y* = , where *k* is a constant.

When *x* is inversely proportional to *y* the graph is the same shape as the graph of *y* = 

**Examples**

**Example 1** *y* is directly proportional to *x*.  
When *y* = 16, *x* = 5.  
**a** Find *x* when *y* = 30.  
**b** Sketch the graph of the formula.

|  |  |
| --- | --- |
| **a**  *y* = *kx*  16 = *k* × 5  *k* = 3.2  *y* = 3.2*x*  When *y* = 30,  30 = 3.2 × *x*  *x* = 9.375 | **1** Write *y* is directly proportional to *x*, using the symbol .  **2** Write the equation using *k*.  **3** Substitute *y* = 16 and *x* = 5 into  *y* = *kx*.  **4** Solve the equation to find *k*.  **5** Substitute the value of *k* back into the equation *y* = *kx*.  **6** Substitute *y* = 30 into *y* = 3.2*x* and solve to find *x* when *y* = 30. |

|  |  |
| --- | --- |
| A black and white image of a cross section  AI-generated content may be incorrect.**b** | **7** The graph of *y* = 3.2*x* is a straight line passing through (0, 0) with a gradient of 3.2. |

**Example 2** *y* is directly proportional to *x*2.  
When *x* = 3, *y* = 45.  
**a** Find *y* when *x* = 5.  
**b** Find *x* when *y* = 20.

|  |  |
| --- | --- |
| **a**  *y* = *kx*2  45 = *k* × 32  *k* = 5  *y* = 5*x*2  When *x* = 5,  *y* = 5 × 52  *y* = 125  **b** 20 = 5 × *x*2  *x*2 = 4  *x* = ±2 | **1** Write *y* is directly proportional to *x*2, using the symbol .  **2** Write the equation using *k*.  **3** Substitute *y* = 45 and *x* = 3 into  *y* = *kx*2.  **4** Solve the equation to find *k*.  **5** Substitute the value of *k* back into the equation *y* = *kx*2.  **6** Substitute *x* = 5 into *y* = 5*x*2 and solve to find *y* when *x* = 5.  **7** Substitute *y* = 20 into *y* = 5*x*2 and solve to find *x* when *y* = 4. |

**Example 3** *P* is inversely proportional to *Q*.  
When *P* = 100, *Q* = 10.  
Find *Q* when *P* = 20.

|  |  |
| --- | --- |
| *k* = 1000 | **1** Write *P* is inversely proportional  to *Q*, using the symbol .  **2** Write the equation using *k*.  **3** Substitute *P* = 100 and *Q* = 10.    **4** Solve the equation to find *k*.  **5** Substitute the value of *k* into  **6** Substitute *P* = 20 into  and solve to find *Q* when *P* = 20. |

**Practice**

**Hint**

Substitute the values given for *P* and *h* into the formula to calculate *k*.

**1** Paul gets paid an hourly rate. The amount of pay (£*P*) is directly proportional to the number of hours (*h*) he works.   
When he works 8 hours he is paid £56.  
If Paul works for 11 hours, how much is he paid?

**2** *x* is directly proportional to *y*. *x* = 35 when *y* = 5.

**a** Find a formula for *x* in terms of *y*.

**b** Sketch the graph of the formula.

**c** Find *x* when *y* = 13.

**d** Find *y* when *x* = 63.

**3** *Q* is directly proportional to the square of *Z*.   
 *Q* = 48 when *Z* = 4.

**a** Find a formula for *Q* in terms of *Z*.

**b** Sketch the graph of the formula.

**c** Find *Q* when *Z* = 5.

**d** Find *Z* when *Q* = 300.

**4** *y* is directly proportional to the square of *x*.  
 *x* = 2 when *y* = 10.

**a** Find a formula for *y* in terms of *x*.

**b** Sketch the graph of the formula.

**c** Find *x* when *y* = 90.

**5** *B* is directly proportional to the square root of *C.   
 C* = 25 when *B* = 10.

**a** Find *B* when *C* = 64.

**b** Find *C* when *B* = 20.

**6** *C* is directly proportional to *D*.  
 *C* = 100 when *D* = 150.  
 Find *C* when *D* = 450.

**7** *y* is directly proportional to *x*.  
 *x* = 27 when *y* = 9.  
 Find *x* when *y* = 3.7.

**8** *m* is proportional to the cube of *n*.  
 *m* = 54 when *n* = 3.  
 Find *n* when *m* = 250.

**Extend**

**9** *s* is inversely proportional to *t*.

**a** Given that *s* = 2 when *t* = 2, find a formula for *s* in terms of *t*.

**b** Sketch the graph of the formula.

**c** Find *t* when *s* = 1.

**10** *a* is inversely proportional to *b*.  
*a* = 5 when *b* = 20.

**a** Find *a* when *b* = 50.

**b** Find *b* when *a* = 10.

**11** *v* is inversely proportional to *w*.  
*w* = 4 when *v* = 20.

**a** Find a formula for *v* in terms of *w*.

**b** Sketch the graph of the formula.

**c** Find *w* when *v* = 2.

**12** *L* is inversely proportional to *W*.   
*L* = 12 when *W* = 3.  
Find *W* when *L* = 6.

**13** *s* is inversely proportional to *t*.  
*s* = 6 when *t* = 12.

**a** Find *s* when *t* = 3.

**b** Find *t* when *s* = 18.

**14** *y* is inversely proportional to *x*2.  
*y* = 4 when *x* = 2.  
Find *y* when *x* = 4.

**15** *y* is inversely proportional to the square root of *x*.  
*x* = 25 when *y* = 1.  
Find *x* when *y* = 5.

**16** *a* is inversely proportional to *b*.  
*a* = 0.05 when *b* = 4.

**a** Find *a* when *b* = 2.

**b** Find *b* when *a* = 2.

**Answers**

**1** £77

A math equation with a couple of lines

AI-generated content may be incorrect.**2 a** *x* = 7*y* **b**

**c** 91 **d** 9

A graph of a function

AI-generated content may be incorrect.**3 a** *Q* = 3*Z*2 **b**

**c** 75 **d** ±10

A graph of a function

AI-generated content may be incorrect.**4 a** *y* = 2.5*x*2 **b**

**c** ±6

**5 a** 16 **b** 100

**6** 300

**7** 11.1

**8** 5

A graph of a function

AI-generated content may be incorrect.**9 a** **  **b**

**c** 4

**10 a** 2 **b** 10

A graph of a function

AI-generated content may be incorrect.**11 a** ** **b**

**c** 40

**12** 6

**13 a** 24 **b** 4

**14** 1

**15** 1

**16 a** 0.1 **b** 0.1

**Circle theorems**

**A LEVEL LINKS**

**Scheme of work:** 2b. Circles – equation of a circle, geometric problems on a grid

**A circle with a line in the center

AI-generated content may be incorrect.Key points**

A chord is a straight line joining two points on the circumference of a circle.  
So AB is a chord.

A circle with a line in the center

AI-generated content may be incorrect.

A tangent is a straight line that touches the circumference of a circle at only one point.  
The angle between a tangent and the radius is 90°.

A diagram of a circle with lines and a triangle

AI-generated content may be incorrect.

Two tangents on a circle that meet at a point outside the circle are equal in length.  
So AC = BC.

A circle with a triangle in it

AI-generated content may be incorrect.

The angle in a semicircle is a right angle.  
So angle ABC = 90°.

A circle with a triangle and a triangle in the center

AI-generated content may be incorrect.When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.  
So angle AOB = 2 × angle ACB.

A circle with a triangle in it

AI-generated content may be incorrect.Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.   
So angle ACB = angle ADB and   
angle CAD = angle CBD.

A square with circles and lines in a circle

AI-generated content may be incorrect.A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle.  
Opposite angles in a cyclic quadrilateral total 180°.  
So *x* + *y* = 180° and *p* + *q* = 180°.

A circle with a triangle and a triangle in the center

AI-generated content may be incorrect.The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem.  
So angle BAT = angle ACB.

**A drawing of a triangle with circles and a circle

AI-generated content may be incorrect.Examples**

**Example 1** Work out the size of each angle   
marked with a letter.  
Give reasons for your answers.

|  |  |
| --- | --- |
| Angle *a* = 360° − 92°  = 268°  as the angles in a full turn total 360°.  Angle *b* = 268° ÷ 2  = 134° as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference. | **1** The angles in a full turn total 360°.  **2** Angles *a* and *b* are subtended by  the same arc, so angle *b* is half of angle *a*. |

A circle with a triangle in it

AI-generated content may be incorrect.

**Example 2** Work out the size of the angles in the triangle.  
 Give reasons for your answers.

|  |  |
| --- | --- |
| Angles are 90°, 2*c* and *c*.  90° + 2*c* + *c* = 180°  90° + 3*c* = 180°  3*c* = 90°  *c* = 30°  2*c* = 60°  The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°. | **1** The angle in a semicircle is a right angle.  **2** Angles in a triangle total 180°.  **3** Simplify and solve the equation. |

A circle with a triangle and a cross

AI-generated content may be incorrect.

**Example 3** Work out the size of each angle marked with a letter.  
 Give reasons for your answers.

|  |  |
| --- | --- |
| Angle *d* = 55° as angles subtended by the same arc are equal.  Angle *e* = 28° as angles subtended by the same arc are equal. | **1** Angles subtended by the same arc are equal so angle 55° and angle *d* are equal.  **2** Angles subtended by the same arc are equal so angle 28° and angle *e* are equal. |

A circle with a triangle and a circle with a line in the middle

AI-generated content may be incorrect.

**Example 4** Work out the size of each angle marked with a letter.  
 Give reasons for your answers.

|  |  |
| --- | --- |
| Angle *f* = 180° − 94°  = 86°  as opposite angles in a cyclic quadrilateral total 180°. | **1** Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle *f* total 180°.  *(continued on next page)* |
| Angle *g* = 180° − 86°  = 84°  as angles on a straight line total 180°.  Angle *h* = angle *f* = 86° as angles subtended by the same arc are equal. | **2** Angles on a straight line total 180° so angle *f* and angle *g* total 180°.  **3** Angles subtended by the same arc are equal so angle *f* and angle *h* are equal. |

A diagram of a triangle with a circle and a circle with a triangle in the center

AI-generated content may be incorrect.**Example 5** Work out the size of each angle marked with a letter.  
 Give reasons for your answers.

|  |  |
| --- | --- |
| Angle *i* = 53° because of the alternate segment theorem.  Angle *j* = 53° because it is the alternate angle to 53°.  Angle *k* = 180° − 53° − 53°  = 74°  as angles in a triangle total 180°. | **1** The angle between a tangent and chord is equal to the angle in the alternate segment.  **2** As there are two parallel lines, angle 53° is equal to angle *j* because they are alternate angles.  **3** The angles in a triangle total 180°, so *i* + *j* + *k* = 180°. |

A black and white drawing of a circle and a triangle

AI-generated content may be incorrect.

**Example 6** XZ and YZ are two tangents to a circle with centre O.  
 Prove that triangles XZO and YZO are congruent.

|  |  |
| --- | --- |
| Angle OXZ = 90° and angle OYZ = 90° as the angles in a semicircle are right angles.  OZ is a common line and is the hypotenuse in both triangles.  OX = OY as they are radii of the same circle.  So triangles XZO and YZO are congruent, RHS. | For two triangles to be congruent you need to show one of the following.   * All three corresponding sides are equal (SSS). * Two corresponding sides and the included angle are equal (SAS). * One side and two corresponding angles are equal (ASA). * A right angle, hypotenuse and a shorter side are equal (RHS). |

**Practice**

**1** Work out the size of each angle marked with a letter.

A circle with a triangle and a triangle in the center

AI-generated content may be incorrect. Give reasons for your answers.

**a b**

A drawing of a circle and a circle with a circle and a circle with a circle and a circle with a circle and a circle with a circle and a circle with a circle and a circle with

AI-generated content may be incorrect.

A circle with a circle and a circle with a circle with a circle and a circle with a circle with a circle with a circle with a circle with a circle with a circle with a circle with

AI-generated content may be incorrect.A circle with a triangle and a circle with a triangle and a circle with a circle with a circle with a circle with a circle with a circle with a circle with a circle with a circle with

AI-generated content may be incorrect.

**C d**

A diagram of a circle with circles and lines

AI-generated content may be incorrect. **e**

**2** Work out the size of each angle marked with a letter.  
 Give reasons for your answers.

A circle with circles and a triangle in the center

AI-generated content may be incorrect. **a b**

A circle with circles and a triangle in it

AI-generated content may be incorrect.

A circle with a triangle and numbers in it

AI-generated content may be incorrect. **c**

**Hint**

The reflex angle at point O and angle *g* are subtended by the same arc. So the reflex angle is twice the size of angle *g*.

A circle with a triangle and a triangle in the center

AI-generated content may be incorrect. **d**

**Hint**

Angle 18° and angle *h* are subtended by the same arc.

A black and white circle with a triangle and a triangle with numbers

AI-generated content may be incorrect.A triangle with a triangle in the middle

AI-generated content may be incorrect.**3** Work out the size of each angle marked with a letter.  
 Give reasons for your answers.

**a b**

A circle with triangles and lines

AI-generated content may be incorrect.A circle with triangles and lines

AI-generated content may be incorrect.

**Hint**

One of the angles is in a semicircle.

**C d**

**4** Work out the size of each angle marked with a letter.  
 Give reasons for your answers.

A black circle with white circles and numbers on it

AI-generated content may be incorrect. **a**

**Hint**

An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

A circle with a triangle in the center

AI-generated content may be incorrect.A diagram of a triangle with circles and lines

AI-generated content may be incorrect. **b c**

A black and white drawing of a triangle and a circle

AI-generated content may be incorrect. **d**

**Hint**

One of the angles is in a semicircle.

**Extend**

**5** Prove the alternate segment theorem.

**Answers**

**1 a** *a* = 112°, angle OAP = angle OBP = 90° and angles in a quadrilateral total 360°.

**b** *b* = 66°, triangle OAB is isosceles, Angle OAP = 90° as AP is tangent to the circle.

**c** *c* = 126°, triangle OAB is isosceles.  
 *d* = 63°, Angle OBP = 90° as BP is tangent to the circle.

**d** *e* = 44°, the triangle is isosceles, so angles *e* and angle OBA are equal. The angle OBP = 90° as BP is tangent to the circle.  
 *f* = 92°, the triangle is isosceles.

**e** *g* = 62°, triangle ABP is isosceles as AP and BP are both tangents to the circle.  
 *h* = 28°, the angle OBP = 90°.

**2 a** *a* = 130°, angles in a full turn total 360°.  
 *b* = 65°, the angle at the centre of a circle is twice the angle at the circumference.  
 *c* = 115°, opposite angles in a cyclic quadrilateral total 180°.

**b** *d* = 36°, isosceles triangle.  
 *e* = 108°, angles in a triangle total 180°.  
 *f* = 54°, angle in a semicircle is 90°.

**c** *g* = 127°, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.

**d** *h* = 36°, the angle at the centre of a circle is twice the angle at the circumference.

**3 a** *a* = 25°, angles in the same segment are equal.  
 *b* = 45°, angles in the same segment are equal.

**b** *c* = 44°, angles in the same segment are equal.  
 *d* = 46°, the angle in a semicircle is 90° and the angles in a triangle total 180°.

**c** *e* = 48°, the angle at the centre of a circle is twice the angle at the circumference.  
 *f* = 48°, angles in the same segment are equal.

**d** *g* = 100°, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.  
 *h* = 100°, angles in the same segment are equal.

**4 a** *a* = 75°, opposite angles in a cyclic quadrilateral total 180°.  
 *b* = 105°, angles on a straight line total 180°.  
 *c* = 94°, opposite angles in a cyclic quadrilateral total 180°.

**b** *d* = 92°, opposite angles in a cyclic quadrilateral total 180°.  
 *e* = 88°, angles on a straight line total 180°.  
 *f* = 92°, angles in the same segment are equal.

**c** *h* = 80°, alternate segment theorem.

**d** *g* = 35°, alternate segment theorem and the angle in a semicircle is 90°.

**5** Angle BAT = *x*.

Angle OAB = 90° − *x* because the angle between the tangent and the radius is 90°.

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = 180° − (90° − *x*) − (90° − *x*) = 2*x* because angles in a triangle total 180°.

Angle ACB = 2*x* ÷ 2 = *x* because the angle at the centre is twice the angle at the circumference.

A diagram of a triangle with a circle and a circle with a triangle

AI-generated content may be incorrect.

**Trigonometry in right-angled triangles**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**A diagram of a triangle

AI-generated content may be incorrect.Key points**

* In a right-angled triangle:
* the side opposite the right angle is called the hypotenuse
* the side opposite the angle *θ* is called the opposite
* the side next to the angle *θ* is called the adjacent.
* In a right-angled triangle:
  + the ratio of the opposite side to the hypotenuse is the sine of angle *θ*, 
  + the ratio of the adjacent side to the hypotenuse is the cosine of angle *θ*, 
  + the ratio of the opposite side to the adjacent side is the tangent of angle *θ*, 
* If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: sin−1, cos−1, tan−1.
* The sine, cosine and tangent of some angles may be written exactly.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **0** | **30°** | **45°** | **60°** | **90°** |
| **sin** | 0 |  |  |  | 1 |
| **cos** | 1 |  |  |  | 0 |
| **tan** | 0 |  | 1 |  |  |

**A triangle with text and numbers

AI-generated content may be incorrect.Examples**

**Example 1** Calculate the length of side *x*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with text on it  AI-generated content may be incorrect.        *x* = 6.620 267 5...  *x* = 6.62 cm | **1** Always start by labelling the sides.  **2** You are given the adjacent and the hypotenuse so use the cosine ratio.  **3** Substitute the sides and angle into the cosine ratio.  **4** Rearrange to make *x* the subject.  **5** Use your calculator to work out  6 ÷ cos 25°.  **6** Round your answer to 3 significant figures and write the units in your answer. |

A drawing of a triangle with a circle and a circle

AI-generated content may be incorrect.

**Example 2** Calculate the size of angle *x*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with text on it  AI-generated content may be incorrect.      *x* = tan–1  *x* = 33.690 067 5...  *x* = 33.7° | **1** Always start by labelling the sides.  **2** You are given the opposite and the adjacent so use the tangent ratio.  **3** Substitute the sides and angle into the tangent ratio.  **4** Use tan−1 to find the angle.  **5** Use your calculator to work out  tan–1(3 ÷ 4.5).  **6** Round your answer to 3 significant figures and write the units in your answer. |

A black triangle with white text

AI-generated content may be incorrect.**Example 3** Calculate the exact size of angle *x*.

|  |  |
| --- | --- |
| A triangle with text on it  AI-generated content may be incorrect.      *x* = 30° | **1** Always start by labelling the sides.  **2** You are given the opposite and the adjacent so use the tangent ratio.  **3** Substitute the sides and angle into the tangent ratio.  **4** Use the table from the key points to find the angle. |

**Practice**

**1** Calculate the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A triangle with text on it

AI-generated content may be incorrect.A black triangle with white text

AI-generated content may be incorrect. **a b**

A black line drawing of a triangle

AI-generated content may be incorrect.A black triangle with black text

AI-generated content may be incorrect. **c d**

A black and white triangle with text

AI-generated content may be incorrect.A black triangle with white text

AI-generated content may be incorrect. **e f**

A black triangle with text and numbers

AI-generated content may be incorrect.**2** Calculate the size of angle *x* in each triangle.  
 Give your answers correct to 1 decimal place.

A triangle with text on it

AI-generated content may be incorrect. **a b**

A black triangle with a black line

AI-generated content may be incorrect.

A black and white drawing of a triangle

AI-generated content may be incorrect. **c d**

A triangle with numbers and a triangle in the middle

AI-generated content may be incorrect.**3** Work out the height of the isosceles triangle.  
 Give your answer correct to 3 significant figures.

**Hint:**

Split the triangle into two right-angled triangles.

A triangle with a square in the middle

AI-generated content may be incorrect.**4** Calculate the size of angle *θ*.  
 Give your answer correct to 1 decimal place.

**Hint:**

First work out the length of the common side to both triangles, leaving your answer in surd form.

A triangle with square and square

AI-generated content may be incorrect.A black triangle with white text

AI-generated content may be incorrect.**5** Find the exact value of *x* in each triangle.

**a b**

A triangle with text and numbers

AI-generated content may be incorrect.

A black line with white text

AI-generated content may be incorrect. **c d**

**The cosine rule**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:**Pure Year 1, 9.1 The cosine rule

**A triangle with letters and numbers

AI-generated content may be incorrect.Key points**

* *a* is the side opposite angle A.

*b* is the side opposite angle B.

*c* is the side opposite angle C.

* You can use the cosine rule to find the length of a side when two sides and the included angle are given.
* To calculate an unknown side use the formula .
* Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
* To calculate an unknown angle use the formula .

**A triangle with a circle and a circle with letters

AI-generated content may be incorrect.Examples**

**Example 4** Work out the length of side *w*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with text on it  AI-generated content may be incorrect.      *w*2 = 33.804 040 51...  *w* =  *w* = 5.81 cm | **1** Always start by labelling the angles and sides.  **2** Write the cosine rule to find the side.  **3** Substitute the values *a*, *b* and *A* into the formula.  **4** Use a calculator to find *w*2 and  then *w*.  **5** Round your final answer to 3 significant figures and write the units in your answer. |

A triangle with a circle and a circle with the same size

AI-generated content may be incorrect.**Example 5** Work out the size of angle *θ*.  
 Give your answer correct to 1 decimal place.

|  |  |
| --- | --- |
| A triangle with numbers and a circle  AI-generated content may be incorrect.        *θ* = 122.878 349...  *θ* = 122.9° | **1** Always start by labelling the angles and sides.  **2** Write the cosine rule to find the angle.  **3** Substitute the values *a*, *b* and *c* into the formula.  **4** Use cos−1 to find the angle.  **5** Use your calculator to work out  cos–1(–76 ÷ 140).  **6** Round your answer to 1 decimal place and write the units in your answer. |

**Practice**

**6** Work out the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A black triangle with white text

AI-generated content may be incorrect. **a b**

A black and white triangle with a circle and black text

AI-generated content may be incorrect.A black triangle with white text

AI-generated content may be incorrect.

A triangle with text on it

AI-generated content may be incorrect. **c d**

A triangle with numbers and a line

AI-generated content may be incorrect.**7** Calculate the angles labelled *θ* in each triangle.  
 Give your answer correct to 1 decimal place.

A black triangle with white text

AI-generated content may be incorrect. **a b**

A triangle with a circle and a circle

AI-generated content may be incorrect.A drawing of a triangle with numbers and a circle

AI-generated content may be incorrect. **c d**

A triangle with a number of degrees

AI-generated content may be incorrect.**8 a** Work out the length of WY.  
 Give your answer correct to   
 3 significant figures.

**b** Work out the size of angle WXY.  
 Give your answer correct to   
 1 decimal place.

**The sine rule**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:**Pure Year 1, 9.2 The sine rule

**A triangle with letters and numbers

AI-generated content may be incorrect.Key points**

* *a* is the side opposite angle A.  
  *b* is the side opposite angle B.  
  *c* is the side opposite angle C.
* You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
* To calculate an unknown side use the formula .
* Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
* To calculate an unknown angle use the formula .

**A triangle with numbers and a circle

AI-generated content may be incorrect.Examples**

**Example 6** Work out the length of side *x*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with numbers and a few circles  AI-generated content may be incorrect.        *x* = 6.09 cm | **1** Always start by labelling the angles and sides.  **2** Write the sine rule to find the side.  **3** Substitute the values *a*, *b*, *A* and *B* into the formula.  **4** Rearrange to make *x* the subject.  **5** Round your answer to 3 significant figures and write the units in your answer. |

A triangle with a circle and numbers

AI-generated content may be incorrect.**Example 7** Work out the size of angle *θ*.  
 Give your answer correct to 1 decimal place.

|  |  |
| --- | --- |
| A triangle with a circle and a circle in the center  AI-generated content may be incorrect.        *θ* = 27.2° | **1** Always start by labelling the angles and sides.  **2** Write the sine rule to find the angle.  **3** Substitute the values *a*, *b*, *A* and *B* into the formula.  **4** Rearrange to make sin *θ* the subject.  **5** Use sin−1 to find the angle. Round your answer to 1 decimal place and write the units in your answer. |

**Practice**

**9** Find the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A triangle with numbers and a few circles

AI-generated content may be incorrect.A triangle with numbers and a few letters

AI-generated content may be incorrect.

**a b**

A triangle with numbers and circles

AI-generated content may be incorrect.

**c d**

A black triangle with white text

AI-generated content may be incorrect.

**10** Calculate the angles labelled *θ* in each triangle.  
 Give your answer correct to 1 decimal place.

A black and white image of a triangle with white text

AI-generated content may be incorrect.A triangle with a circle and circles

AI-generated content may be incorrect.

**a b**

A triangle with a circle and a circle in the middle

AI-generated content may be incorrect.

A triangle with numbers and a circle

AI-generated content may be incorrect. **c d**

A black and white triangle with numbers and a circle

AI-generated content may be incorrect.**11** **a** Work out the length of QS.  
 Give your answer correct to 3 significant figures.

**b** Work out the size of angle RQS.  
 Give your answer correct to 1 decimal place.

**Areas of triangles**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:**Pure Year 1, 9.3 Areas of triangles

**A triangle with text on it

AI-generated content may be incorrect.Key points**

* *a* is the side opposite angle A.  
  *b* is the side opposite angle B.  
  *c* is the side opposite angle C.
* The area of the triangle is .

**A triangle with numbers and a circle

AI-generated content may be incorrect.Examples**

**Example 8** Find the area of the triangle.

|  |  |
| --- | --- |
| A triangle with text on it  AI-generated content may be incorrect.  Area =  Area =  Area = 19.805 361...  Area = 19.8 cm2 | **1** Always start by labelling the sides and angles of the triangle.  **2** State the formula for the area of a triangle.  **3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.  **4** Use a calculator to find the area.  **5** Round your answer to 3 significant figures and write the units in your answer. |

**Practice**

**12** Work out the area of each triangle.  
 Give your answers correct to 3 significant figures.

A black triangle with white text

AI-generated content may be incorrect.A black triangle with white text

AI-generated content may be incorrect. **a b**

A triangle with numbers and a circle

AI-generated content may be incorrect. **c**

A triangle with text on it

AI-generated content may be incorrect.

**13** The area of triangle XYZ is 13.3 cm2.  
 Work out the length of XZ.

**Hint:**

Rearrange the formula to make a side the subject.

**Extend**

**14** Find the size of each lettered angle or side.  
 Give your answers correct to 3 significant figures.

**Hint:**

For each one, decide whether to use the cosine or sine rule.

A black triangle with black text

AI-generated content may be incorrect.A triangle with numbers and a circle

AI-generated content may be incorrect. **a b**

A triangle with a circle and numbers

AI-generated content may be incorrect.**c d**

A black line with black text

AI-generated content may be incorrect.

A triangle with text on it

AI-generated content may be incorrect.**15** The area of triangle ABC is 86.7 cm2.  
 Work out the length of BC.  
 Give your answer correct to 3 significant figures.

**Answers**

**1 a** 6.49 cm **b** 6.93 cm **c** 2.80 cm   
 **d** 74.3 mm **e** 7.39 cm **f** 6.07 cm

**2 a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°

**3** 5.71 cm

**4** 20.4°

**5 a** 45° **b** 1 cm **c** 30° **d**  cm

**6 a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm

**7 a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°

**8 a** 13.7 cm **b** 76.0°

**9 a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm

**10 a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°

**11 a** 8.13 cm **b** 32.3°

**12 a** 18.1 cm2 **b** 18.7 cm2 **c** 693 mm2

**13** 5.10 cm

**14 a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

**15** 15.3 cm

**Rearranging equations**

**A LEVEL LINKS**

**Scheme of work:** 6a. Definition, differentiating polynomials, second derivatives

**Textbook:**Pure Year 1, 12.1 Gradients of curves

**Key points**

* To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
* You may need to factorise the terms containing the new subject.

**Examples**

**Example 1** Make *t* the subject of the formula *v* = *u* + *at*.

|  |  |
| --- | --- |
| *v* = *u* + *at*  *v* − *u* = *at* | **1** Get the terms containing *t* on one side and everything else on the other side.  **2** Divide throughout by *a*. |

**Example 2** Make *t* the subject of the formula *r* = 2*t* − *πt*.

|  |  |
| --- | --- |
| *r* = 2*t* − *πt*  *r* = *t*(2 − *π*) | **1** All the terms containing *t* are already on one side and everything else is on the other side.  **2** Factorise as *t* is a common factor.  **3** Divide throughout by 2 − *π*. |

**Example 3** Make *t* the subject of the formula .

|  |  |
| --- | --- |
| 2*t* + 2*r* = 15*t*  2*r* = 13*t* | **1** Remove the fractions first by multiplying throughout by 10.  **2** Get the terms containing *t* on one side and everything else on the other side and simplify.  **3** Divide throughout by 13. |

**Example 4** Make *t* the subject of the formula .

|  |  |
| --- | --- |
| *r*(*t* − 1) = 3*t* + 5  *rt* − *r* = 3*t* + 5  *rt* − 3*t* = 5 + *r*  *t*(*r* − 3) = 5 + *r* | **1** Remove the fraction first by multiplying throughout by *t* − 1.  **2** Expand the brackets.  **3** Get the terms containing *t* on one side and everything else on the other side.  **4** Factorise the LHS as *t* is a common factor.  **5** Divide throughout by *r* − 3. |

**Practice**

Change the subject of each formula to the letter given in the brackets.

**1** *C* = *πd*  [*d*]**2** *P* = 2*l* + 2*w* [*w*] **3** *D = * [*T*]

**4** ** [*t*] **5** *u* = *at* – *t* [*t*] **6** *V* = *ax* + 4*x* [*x*]

**7** ** [*y*] **8**  [*a*] **9**  [*d*]

**10**  [*g*] **11** *e*(9 + *x*) = 2*e* + 1 [*e*] **12**  [*x*]

**13** Make *r* the subject of the following formulae.

**a** *A* = *πr*2 **b**  **c** *P* = *πr* + 2*r* **d** 

**14** Make *x* the subject of the following formulae.

**a  b **

**15** Make sin *B* the subject of the formula 

**16** Make cos *B* the subject of the formula *b*2 = *a*2 + *c*2 – 2*ac* cos *B*.

**Extend**

**17** Make *x* the subject of the following equations.

**a**  **b** 

**Answers**

**1** *d* =  **2**  **3** 

**4**  **5**  **6** 

**7** *y* = 2 + 3*x* **8**  **9** 

**10**  **11**  **12** 

**13 a**  **b** 

**c**  **d** 

**14 a**  **b** 

**15** 

**16** 

**17 a**  **b** 

**Volume and surface area of 3D shapes**

**A LEVEL LINKS**

**Scheme of work:** 6b. Gradients, tangents, normals, maxima and minima

A diagram of a rectangular object

AI-generated content may be incorrect.Key points

* Volume of a prism = cross-sectional area × length.
* The surface area of a 3D shape is the total area   
  of all its faces.
* A cylinder with arrows and a straight line

  AI-generated content may be incorrect.Volume of a pyramid =  × area of base × vertical height.
* Volume of a cylinder = *πr*2*h*
* Total surface area of a cylinder = 2*πr*2 + 2*πrh*

A white circle with black arrows

AI-generated content may be incorrect.

* Volume of a sphere = 
* A cone with arrows and letters

  AI-generated content may be incorrect.Surface area of a sphere = 4*πr*2
* Volume of a cone = 
* Total surface area of a cone = *πrl* + *πr*2

A black and white drawing of a triangle

AI-generated content may be incorrect.Examples

**Example 1** The triangular prism has volume 504 cm3.   
 Work out its length.

|  |  |
| --- | --- |
| *V* = *bhl*  504 =  × 9 × 4 × *l*  504 = 18 × *l*  *l* = 504 ÷ 18  = 28 cm | **1** Write out the formula for the volume of a triangular prism.  **2** Substitute known values into the formula.  **3** Simplify  **4** Rearrange to work out *l*.  **5** Remember the units. |

A diagram of a cone with a cone and a cone with a cone and a cone with a cone and a cone with a cone and a cone with a cone and a cone with a cone and

AI-generated content may be incorrect.**Example 2** Calculate the volume of the 3D solid.  
 Give your answer in terms of *π*.

|  |  |
| --- | --- |
| Total volume = volume of hemisphere  + Volume of cone  =  of *πr*3 + *πr*2*h*  Total volume =  ×  × *π ×* 53  +  × *π ×* 52 × 7  = *π*cm3 | **1** The solid is made up of a hemisphere radius 5 cm and  a cone with radius 5 cm and height 12 − 5 = 7 cm.  **2** Substitute the measurements into the formula for the total volume.  **3** Remember the units. |

Practice

**1** Work out the volume of each solid.  
 Leave your answers in terms of *π* where appropriate.

**A diagram of a step

AI-generated content may be incorrect.A rectangular object with a rectangular object in the middle

AI-generated content may be incorrect. a b**

**A cylinder with arrows and a cylinder with a straight line

AI-generated content may be incorrect.A black rectangular object with white text

AI-generated content may be incorrect. c d**

A drawing of a cylinder

AI-generated content may be incorrect. **e** **f**  a sphere with radius 7 cm

**g** a sphere with diameter 9 cm **h** a hemisphere with radius 3 cm

A triangle with a straight line and a straight line

AI-generated content may be incorrect.A cone with a straight line

AI-generated content may be incorrect. **i**  **j**

**2** A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm3.  
 Work out its length.

**A triangle with a straight line and a point

AI-generated content may be incorrect.3** The triangular prism has volume 1768 cm3.  
 Work out its height.

A rectangular object with a rectangular object in the center

AI-generated content may be incorrect.Extend

**4** The diagram shows a solid triangular prism.

All the measurements are in centimetres.

The volume of the prism is *V*cm3.

Find a formula for *V* in terms of *x*.

Give your answer in simplified form.

A rectangular object with numbers and a rectangular object

AI-generated content may be incorrect.**5** The diagram shows the area of each of three   
 faces of a cuboid.

The length of each edge of the cuboid is a whole   
 number of centimetres.

Work out the volume of the cuboid.

**A cylinder with measurements and arrows

AI-generated content may be incorrect.6** The diagram shows a large catering size tin of beans   
 in the shape of a cylinder.

The tin has a radius of 8 cm and a height of 15 cm.

A company wants to make a new size of tin.

The new tin will have a radius of 6.7 cm.

It will have the same volume as the large tin.

Calculate the height of the new tin.

Give your answer correct to one decimal place.

A cylinder with a measurement

AI-generated content may be incorrect.A white circle with black text and black arrows

AI-generated content may be incorrect.**7** The diagram shows a sphere and a solid cylinder.

The sphere has radius 8 cm.

The solid cylinder has a base radius of 4 cm and   
 a height of *h*cm.

The total surface area of the cylinder is half the   
 total surface area of the sphere.

Work out the ratio of the volume of the sphere to  
 the volume of the cylinder.

Give your answer in its simplest form.

A cylinder with a straight line

AI-generated content may be incorrect.**8** The diagram shows a solid metal cylinder.

The cylinder has base radius 4*x* and height 3*x*.

The cylinder is melted down and made into   
 a sphere of radius *r*.

Find an expression for *r* in terms of *x*.

Answers

**1 a** *V* = 396 cm3 **b** *V* = 75 000 cm3

**c** *V* = 402.5 cm3 **d** *V* = 200*π*cm3

**e** *V* = 1008*π* cm3 **f** *V=* *π*  cm3

**g** *V* = 121.5*π* cm3 **h** *V* = 18*π*cm3

**i** *V* = 48*π*cm3 **j** *V* = *π*cm3

**2** 17 cm

**3** 17 cm

**4** *V* = *x*3 + *x*2 + 4*x*

**5** 60 cm3

**6** 21.4 cm

**7** 32 : 9

**8** 

**Area under a graph**

**A LEVEL LINKS**

**Scheme of work:** 7b. Definite integrals and areas under curves

**A graph of a graph with a curve and a chord

AI-generated content may be incorrect.Key points**

* To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium.   
  The area of the trapezium is an approximation for the area under a curve.

A rectangular object with arrows and a rectangular object with a rectangular object in the center

AI-generated content may be incorrect.

* The area of a trapezium = 

**A graph of a curve

AI-generated content may be incorrect.Examples**

**Example 1** Estimate the area of the region between the curve   
*y* = (3 − *x*)(2 + *x*) and the *x*-axis from *x* = 0 to *x* = 3.   
Use three strips of width 1 unit.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | ***x*** | 0 | 1 | 2 | 3 | | ***y* = (3 − *x*)(2 + *x*)** | 6 | 6 | 4 | 0 |   Trapezium 1:  ,  Trapezium 2:  ,  Trapezium 3:  , | **1** Use a table to record the value of *y* on the curve for each value of *x*.  **2** Work out the dimensions of each trapezium. The distances between the *y*-values on the curve and the  *x*-axis give the values for *a*.  *(continued on next page)* |
| Area = 6 + 5 + 2 = 13 units2 | **3** Work out the area of each trapezium. *h* = 1 since the width of each trapezium is 1 unit.  **4** Work out the total area. Remember to give units with your answer. |

A graph of a function

AI-generated content may be incorrect.

**Example 2** Estimate the shaded area.   
 Use three strips of width 2 units.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | ***x*** | 4 | 6 | 8 | 10 | | ***y*** | 7 | 12 | 13 | 4 |  |  |  |  |  |  | | --- | --- | --- | --- | --- | | ***x*** | 4 | 6 | 8 | 10 | | ***y*** | 7 | 6 | 5 | 4 |   Trapezium 1:  ,  Trapezium 2:  ,  Trapezium 3:  ,        Area = 6 + 14 + 8 = 28 units2 | **1** Use a table to record *y* on the curve for each value of *x*.  **2** Use a table to record *y* on the straight line for each value of *x*.  **3** Work out the dimensions of each trapezium. The distances between the *y*-values on the curve and the  *y*-values on the straight line give the values for *a*.  **4** Work out the area of each trapezium. *h* = 2since the width of each trapezium is 2 units.  **5** Work out the total area. Remember to give units with your answer. |

**Practice**

**Hint:**

For a full answer, remember to include ‘units2’.

**1** Estimate the area of the region between the curve *y* = (5 − *x*)(*x* + 2) and the *x*-axis from *x* = 1 to *x* = 5.   
 Use four strips of width 1 unit.

A graph with a curve

AI-generated content may be incorrect.

**2** Estimate the shaded area shown on the axes.   
 Use six strips of width 1 unit.

**3** Estimate the area of the region between the curve *y* = *x*2 − 8*x* + 18 and the *x*-axis   
 from *x* = 2 to *x* = 6.   
 Use four strips of width 1 unit.

A graph with lines and numbers

AI-generated content may be incorrect.**4** Estimate the shaded area.   
 Use six strips of width  unit.

**5** Estimate the area of the region between the curve *y* = −*x*2 − 4*x* + 5 and the   
 *x*-axis from *x* = −5 to *x* = 1.   
 Use six strips of width 1 unit.

A graph with lines and numbers

AI-generated content may be incorrect.

**6** Estimate the shaded area.   
 Use four strips of equal width.

**7** Estimate the area of the region between the curve *y* = −*x*2 + 2*x* + 15 and the   
 *x*-axis from *x* = 2 to *x* = 5.   
 Use six strips of equal width.

A graph of a point

AI-generated content may be incorrect.

**8** Estimate the shaded area.   
 Use seven strips of equal width.

**A graph of a function

AI-generated content may be incorrect.Extend**

**9** The curve *y* = 8*x* − 5 − *x*2 and the line *y* = 2   
 are shown in the sketch.   
 Estimate the shaded area using six strips   
 of equal width.

Graph of a graph on a grid

AI-generated content may be incorrect.**10** Estimate the shaded area using five  
 strips of equal width.

**Answers**

**1** 34 units2

**2** 149 units2

**3** 14 units2

**4** 25 units2

**5** 35 units2

**6** 42 units2

**7** 26 units2

**8** 56 units2

**9** 35 units2

**10** 6 units2