

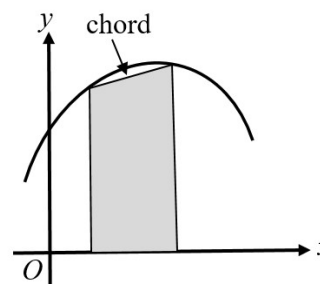
Area under a graph

A LEVEL LINKS

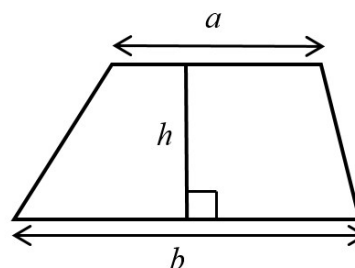
Scheme of work: 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

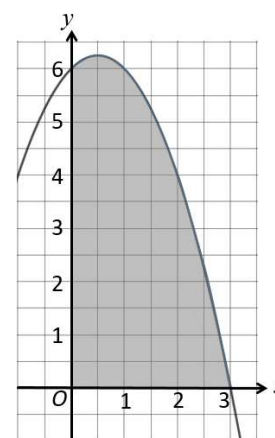


- The area of a trapezium = $\frac{1}{2}h(a+b)$



Examples

Example 1 Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x -axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.



x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
 $a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$

Trapezium 2:
 $a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$

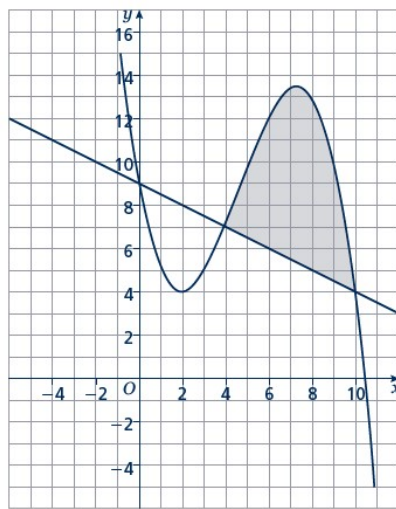
Trapezium 3:
 $a_3 = 4 - 0 = 4$, $b_3 = 0 - 0 = 0$

- 1 Use a table to record the value of y on the curve for each value of x .
- 2 Work out the dimensions of each trapezium. The distances between the y -values on the curve and the x -axis give the values for a .

(continued on next page)

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6 + 6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6 + 4) = 5$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4 + 0) = 2$ <p>Area = 6 + 5 + 2 = 13 units²</p>	<p>3 Work out the area of each trapezium. $h = 1$ since the width of each trapezium is 1 unit.</p> <p>4 Work out the total area. Remember to give units with your answer.</p>
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Example 2 Estimate the shaded area.
Use three strips of width 2 units.



<table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;">6</td><td style="padding: 2px 5px;">8</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;">7</td><td style="padding: 2px 5px;">12</td><td style="padding: 2px 5px;">13</td><td style="padding: 2px 5px;">4</td></tr> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;">6</td><td style="padding: 2px 5px;">8</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;">7</td><td style="padding: 2px 5px;">6</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">4</td></tr> </table> <p>Trapezium 1: $a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$</p> <p>Trapezium 2: $a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$</p> <p>Trapezium 3: $a_3 = 13 - 5 = 8$, $b_3 = 4 - 4 = 0$</p> $\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0 + 6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6 + 8) = 14$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8 + 0) = 8$ <p>Area = 6 + 14 + 8 = 28 units²</p>	x	4	6	8	10	y	7	12	13	4	x	4	6	8	10	y	7	6	5	4	<p>1 Use a table to record y on the curve for each value of x.</p> <p>2 Use a table to record y on the straight line for each value of x.</p> <p>3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.</p> <p>4 Work out the area of each trapezium. $h = 2$ since the width of each trapezium is 2 units.</p> <p>5 Work out the total area. Remember to give units with your answer.</p>
x	4	6	8	10																	
y	7	12	13	4																	
x	4	6	8	10																	
y	7	6	5	4																	

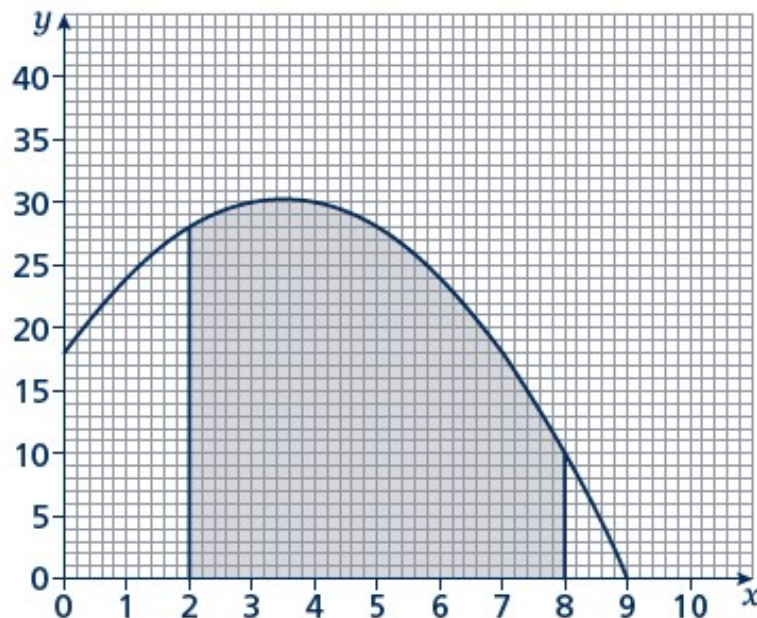
Practice

- 1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x -axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

Hint:

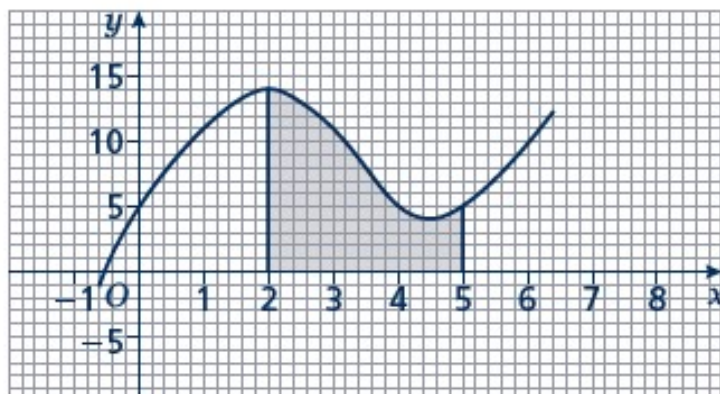
For a full answer, remember to include 'units²'.

- 2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



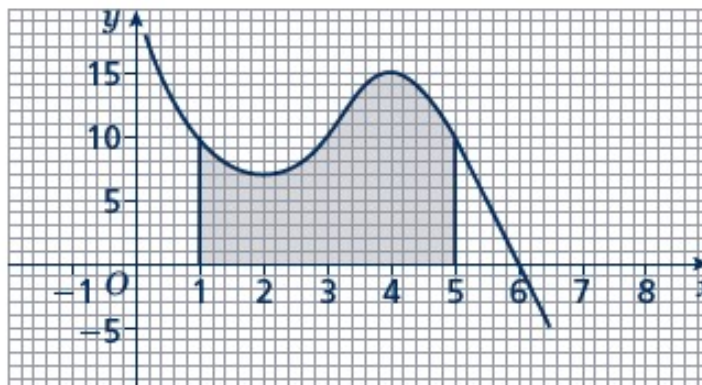
- 3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x -axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

- 4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.



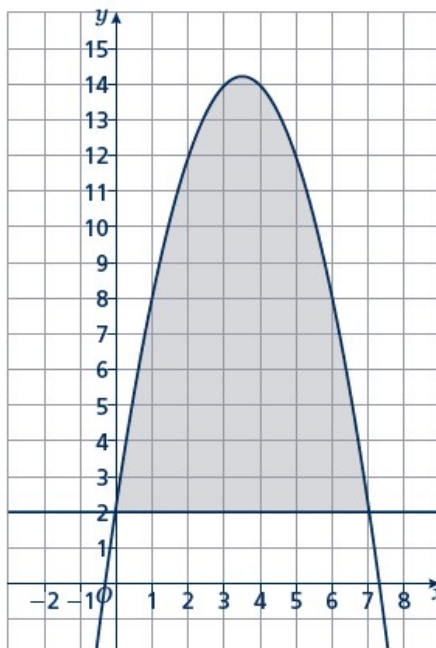
- 5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x -axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

- 6 Estimate the shaded area.
Use four strips of equal width.



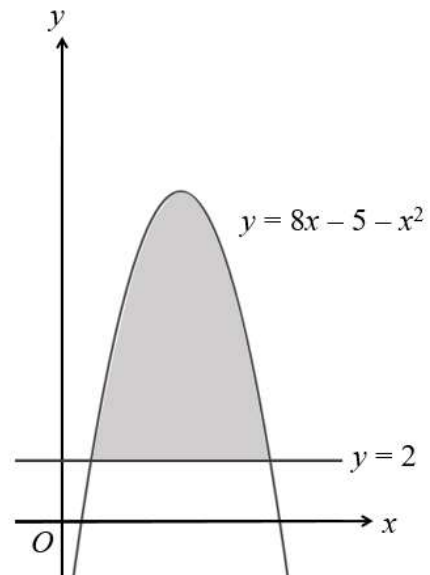
- 7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x -axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

- 8 Estimate the shaded area.
Use seven strips of equal width.

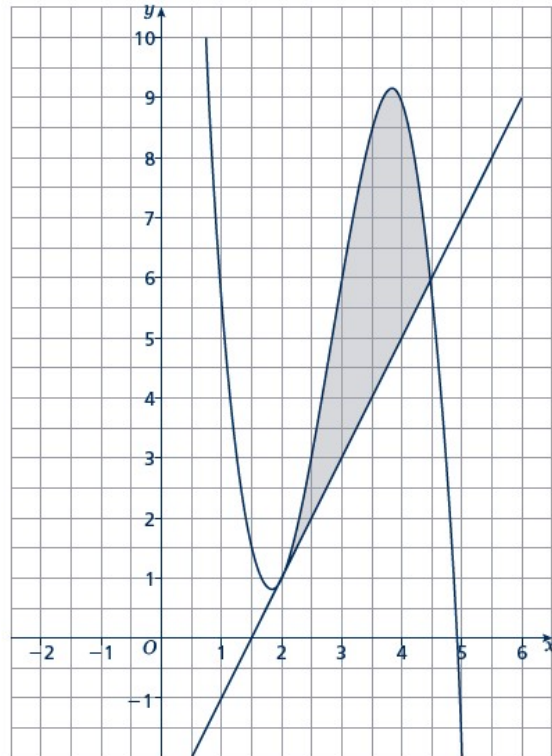


Extend

- 9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch. Estimate the shaded area using six strips of equal width.



- 10 Estimate the shaded area using five strips of equal width.



Answers

1 34 units²

2 149 units²

3 14 units²

4 $25\frac{1}{4}$ units²

5 35 units²

6 42 units²

7 $26\frac{7}{8}$ units²

8 56 units²

9 35 units²

10 $6\frac{1}{4}$ units²