

Rearranging equations

A LEVEL LINKS

Scheme of work: 6a. Definition, differentiating polynomials, second derivatives

Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

| | |
|---|--|
| $v = u + at$ $v - u = at$ $t = \frac{v - u}{a}$ | <ol style="list-style-type: none"> 1 Get the terms containing t on one side and everything else on the other side. 2 Divide throughout by a. |
|---|--|

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

| | |
|---|--|
| $r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$ | <ol style="list-style-type: none"> 1 All the terms containing t are already on one side and everything else is on the other side. 2 Factorise as t is a common factor. 3 Divide throughout by $2 - \pi$. |
|---|--|

Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

| | |
|---|--|
| $\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$ | <ol style="list-style-type: none"> 1 Remove the fractions first by multiplying throughout by 10. 2 Get the terms containing t on one side and everything else on the other side and simplify. 3 Divide throughout by 13. |
|---|--|

Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

| | |
|---|--|
| $r = \frac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t+5$ $rt - 3t = 5+r$ $t(r-3) = 5+r$ $t = \frac{5+r}{r-3}$ | <ol style="list-style-type: none"> 1 Remove the fraction first by multiplying throughout by $t-1$. 2 Expand the brackets. 3 Get the terms containing t on one side and everything else on the other side. 4 Factorise the LHS as t is a common factor. 5 Divide throughout by $r-3$. |
|---|--|

Practice

Change the subject of each formula to the letter given in the brackets.

- | | | |
|--|--|--|
| 1 $C = \pi d$ [d] | 2 $P = 2l + 2w$ [w] | 3 $D = \frac{S}{T}$ [T] |
| 4 $p = \frac{q-r}{t}$ [t] | 5 $u = at - \frac{1}{2}t$ [t] | 6 $V = ax + 4x$ [x] |
| 7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [y] | 8 $x = \frac{2a-1}{3-a}$ [a] | 9 $x = \frac{b-c}{d}$ [d] |
| 10 $h = \frac{7g-9}{2+g}$ [g] | 11 $e(9+x) = 2e+1$ [e] | 12 $y = \frac{2x+3}{4-x}$ [x] |

13 Make r the subject of the following formulae.

| | | | |
|------------------------|-----------------------------------|---------------------------|-------------------------------------|
| a $A = \pi r^2$ | b $V = \frac{4}{3}\pi r^3$ | c $P = \pi r + 2r$ | d $V = \frac{2}{3}\pi r^2 h$ |
|------------------------|-----------------------------------|---------------------------|-------------------------------------|

14 Make x the subject of the following formulae.

| | |
|---|--|
| a $\frac{xy}{z} = \frac{ab}{cd}$ | b $\frac{4\pi cx}{d} = \frac{3z}{py^2}$ |
|---|--|

15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

| | |
|------------------------------------|---|
| a $\frac{p}{q}(sx+t) = x-1$ | b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$ |
|------------------------------------|---|

Answers

1 $d = \frac{C}{\pi}$

2 $w = \frac{P-2l}{2}$

3 $T = \frac{S}{D}$

4 $t = \frac{q-r}{p}$

5 $t = \frac{2u}{2a-1}$

6 $x = \frac{V}{a+4}$

7 $y = 2 + 3x$

8 $a = \frac{3x+1}{x+2}$

9 $d = \frac{b-c}{x}$

10 $g = \frac{2h+9}{7-h}$

11 $e = \frac{1}{x+7}$

12 $x = \frac{4y-3}{2+y}$

13 a $r = \sqrt{\frac{A}{\pi}}$

b $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi+2}$

d $r = \sqrt{\frac{3V}{2\pi h}}$

14 a $x = \frac{abz}{cdy}$

b $x = \frac{3dz}{4\pi cpy^2}$

15 $\sin B = \frac{b \sin A}{a}$

16 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

17 a $x = \frac{q+pt}{q-ps}$

b $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$