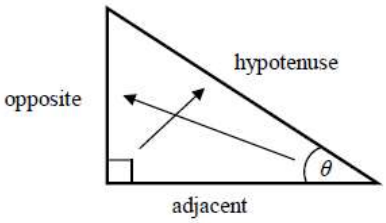
Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

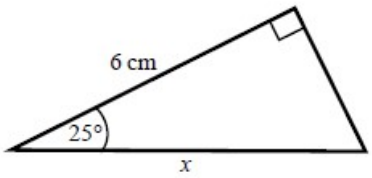
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

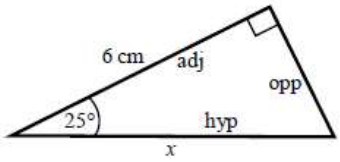
- The sine, cosine and tangent of some angles may be written exactly.

	0	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.





$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 25° = \frac{6}{x}$$

$$x = \frac{6}{\cos 25°}$$

$x = 6.620\,267\,5...$

$x = 6.62$ cm

1 Always start by labelling the sides.

2 You are given the adjacent and the hypotenuse so use the cosine ratio.

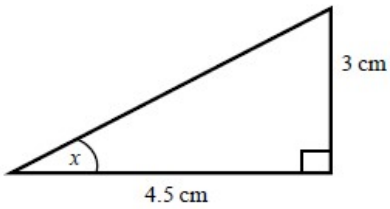
3 Substitute the sides and angle into the cosine ratio.

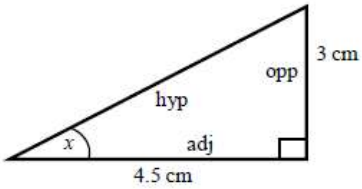
4 Rearrange to make x the subject.

5 Use your calculator to work out $6 \div \cos 25°$.

6 Round your answer to 3 significant figures and write the units in your answer.

Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.





$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{3}{4.5}$$

$$x = \tan^{-1}\left(\frac{3}{4.5}\right)$$

$x = 33.690\,067\,5...$

$x = 33.7°$

1 Always start by labelling the sides.

2 You are given the opposite and the adjacent so use the tangent ratio.

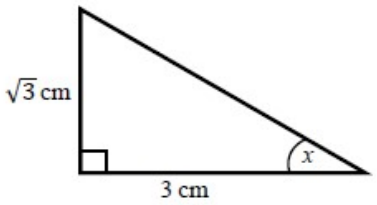
3 Substitute the sides and angle into the tangent ratio.

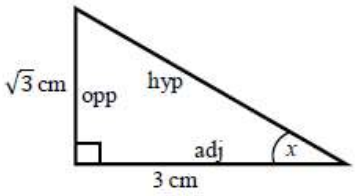
4 Use $\tan^{-1}$ to find the angle.

5 Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.

6 Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle x.





$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{\sqrt{3}}{3}$$

$x = 30°$

1 Always start by labelling the sides.

2 You are given the opposite and the adjacent so use the tangent ratio.

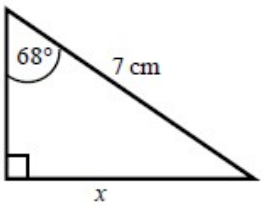
3 Substitute the sides and angle into the tangent ratio.

4 Use the table from the key points to find the angle.

Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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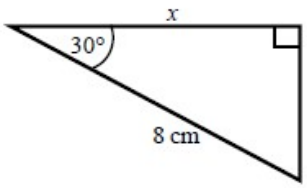

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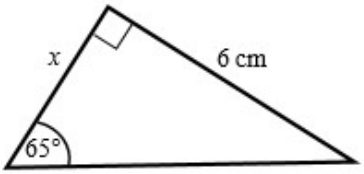

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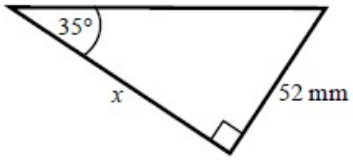

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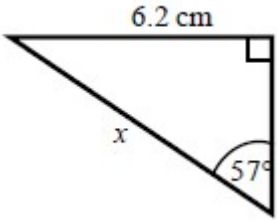

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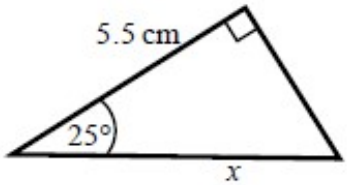
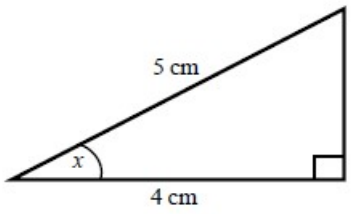


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

a



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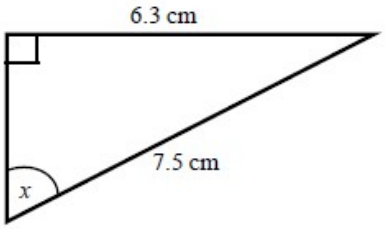

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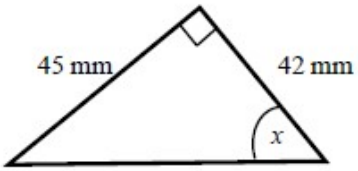

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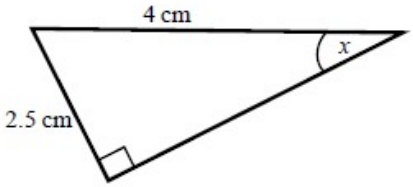
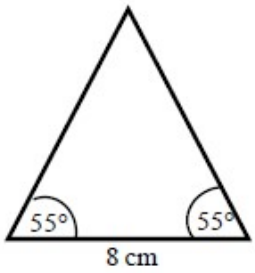


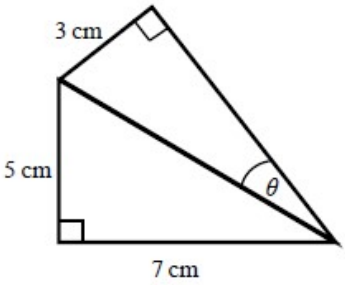
3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



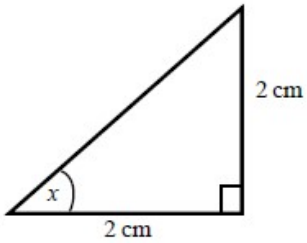
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.



5 Find the exact value of x in each triangle.

a



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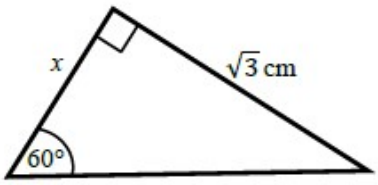

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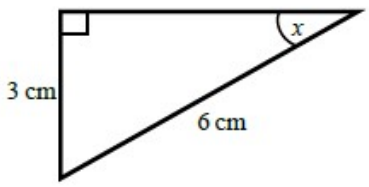

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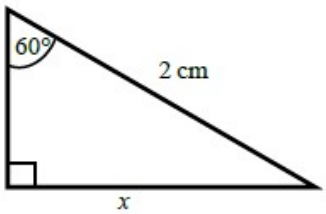
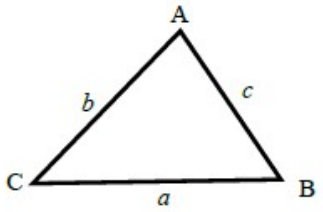


The cosine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

Key points

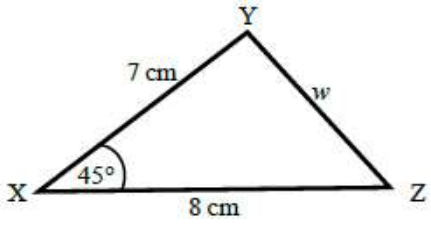
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

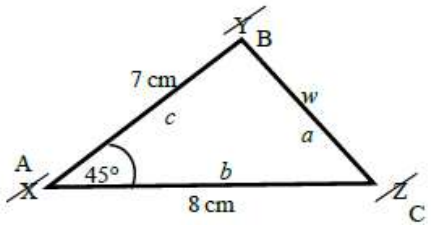


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Examples

Example 4 Work out the length of side w.
Give your answer correct to 3 significant figures.





$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$

$w^2 = 33.804\,040\,51...$
$w = \sqrt{33.80404051}$
$w = 5.81$ cm

1 Always start by labelling the angles and sides.

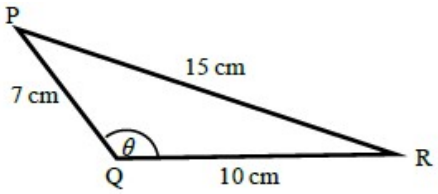
2 Write the cosine rule to find the side.

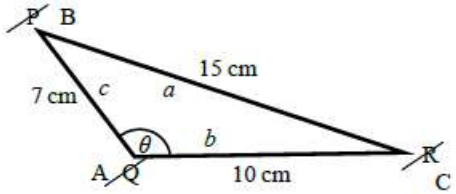
3 Substitute the values a, b and A into the formula.

4 Use a calculator to find w^2 and then w.

5 Round your final answer to 3 significant figures and write the units in your answer.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$$

$$\cos\theta = \frac{-76}{140}$$

$\theta = 122.878\,349...$

$\theta = 122.9°$

1 Always start by labelling the angles and sides.

2 Write the cosine rule to find the angle.

3 Substitute the values a, b and c into the formula.

4 Use $\cos^{-1}$ to find the angle.

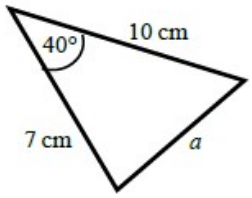
5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$.

6 Round your answer to 1 decimal place and write the units in your answer.

Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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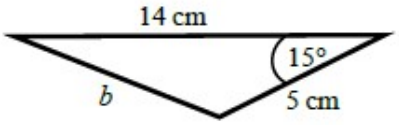

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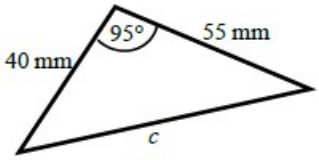

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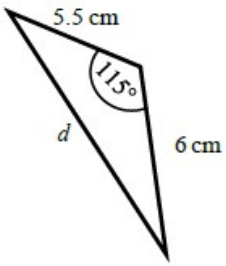
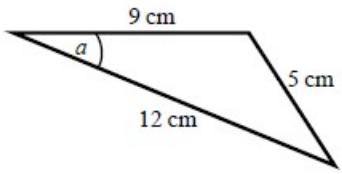


7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



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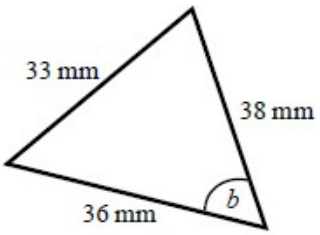

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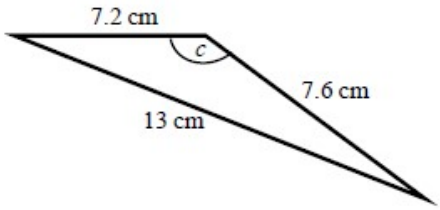

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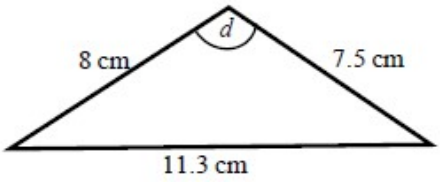
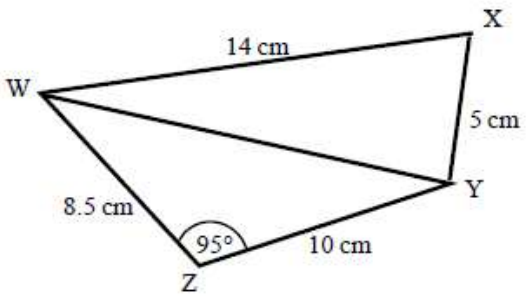


8 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

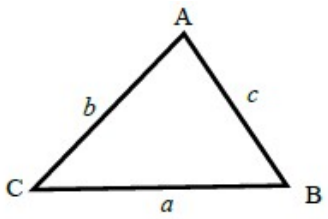


The sine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

Key points

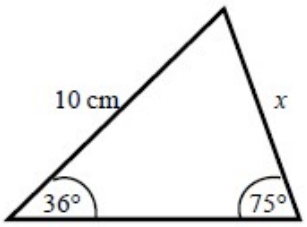
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

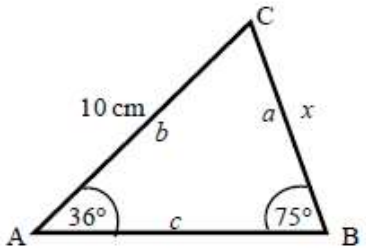


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Examples

Example 6 Work out the length of side x.
Give your answer correct to 3 significant figures.





$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$

$x = \frac{10 \times \sin 36°}{\sin 75°}$

$x = 6.09$ cm

1 Always start by labelling the angles and sides.

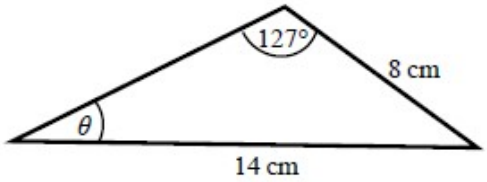
2 Write the sine rule to find the side.

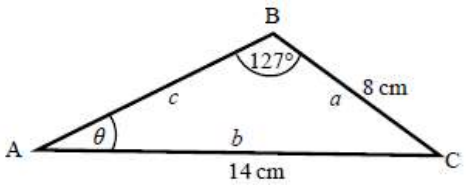
3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make x the subject.

5 Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$$

$$\sin \theta = \frac{8 \times \sin 127°}{14}$$

$$\theta = 27.2°$$

1 Always start by labelling the angles and sides.

2 Write the sine rule to find the angle.

3 Substitute the values a, b, A and B into the formula.

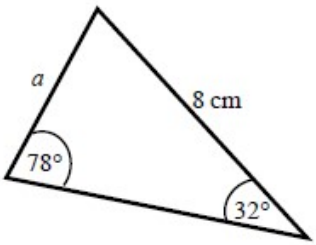
4 Rearrange to make $\sin \theta$ the subject.

5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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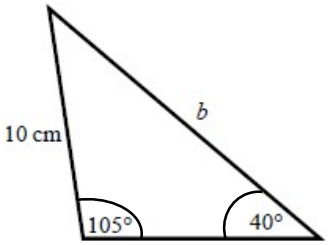
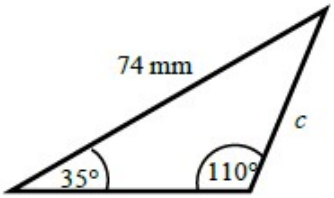


c



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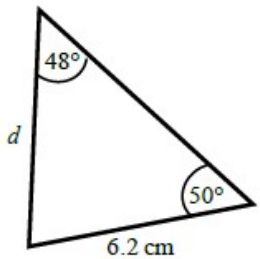
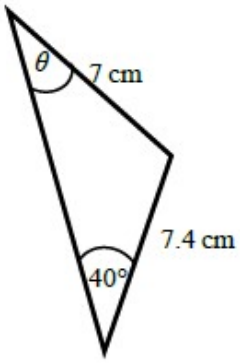
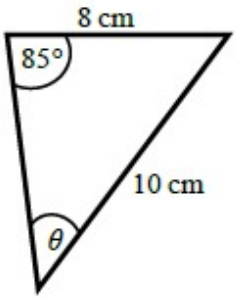


10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



b



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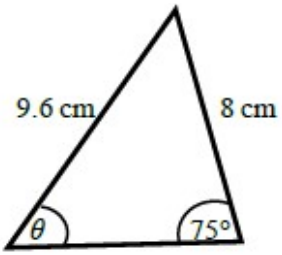

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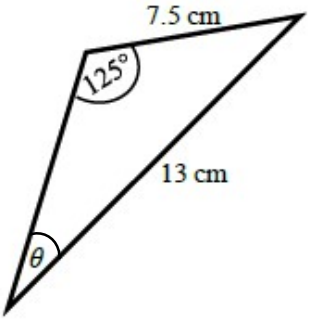
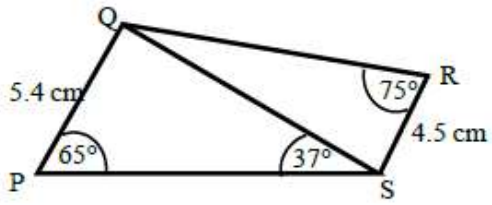


11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

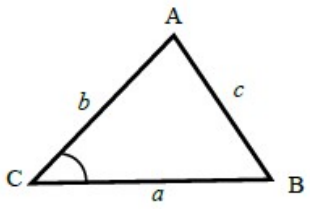


Areas of triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.3 Areas of triangles

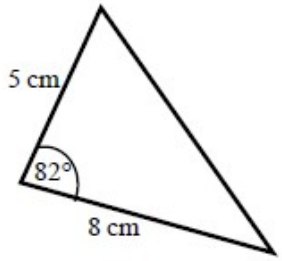
Key points

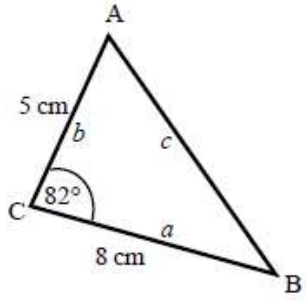
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.





Area = $\frac{1}{2}ab\sin C$

Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$

Area = 19.805 361...

Area = 19.8 cm^2

1 Always start by labelling the sides and angles of the triangle.

2 State the formula for the area of a triangle.

3 Substitute the values of a, b and C into the formula for the area of a triangle.

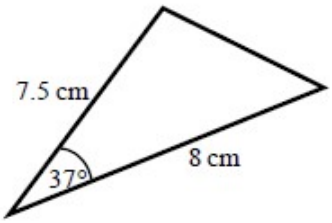
4 Use a calculator to find the area.

5 Round your answer to 3 significant figures and write the units in your answer.

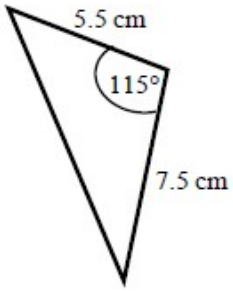
Practice

12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

a



b



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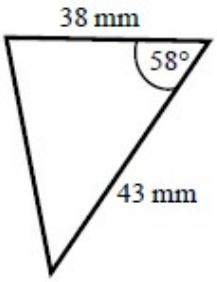
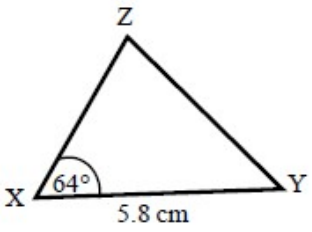


13 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



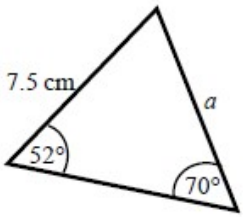
> **Hint:**
>
> Rearrange the formula to make a side the subject.

Extend

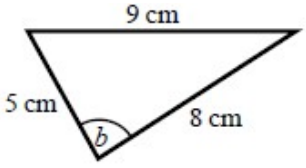
14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

> **Hint:**
>
> For each one, decide whether to use the cosine or sine rule.

a



b



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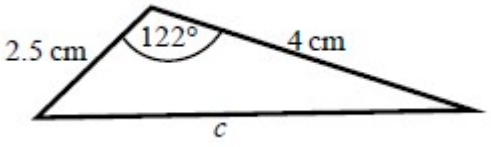
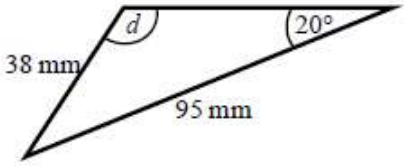
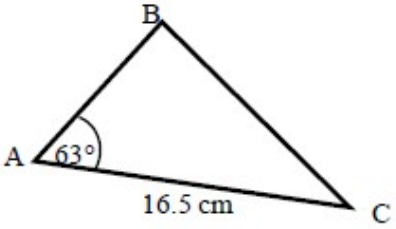


d



15 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

1 **a** 6.49 cm **b** 6.93 cm **c** 2.80 cm
d 74.3 mm **e** 7.39 cm **f** 6.07 cm

2 **a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°

3 5.71 cm

4 20.4°

5 **a** 45° **b** 1 cm **c** 30° **d** $\sqrt{3}$ cm

6 **a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm

7 **a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°

8 **a** 13.7 cm **b** 76.0°

9 **a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm

10 **a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°

11 **a** 8.13 cm **b** 32.3°

12 **a** 18.1 cm^2 **b** 18.7 cm^2 **c** 693 mm^2

13 5.10 cm

14 **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

15 15.3 cm