Parallel and perpendicular lines

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\dfrac{1}{m}$.
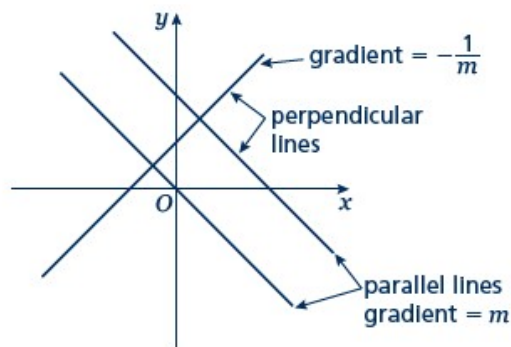
Examples

Example 1   Find the equation of the line parallel to $y = 2x + 4$ which passes through the point $(4, 9)$.

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates into the equation $y = 2x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2   Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\dfrac{1}{m} = -\dfrac{1}{2}$ $y = -\dfrac{1}{2}x + c$ $5 = -\dfrac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\dfrac{1}{2}x + 4$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\dfrac{1}{m}$. **2** Substitute $m = -\dfrac{1}{2}$ into $y = mx + c$. **3** Substitute the coordinates $(-2, 5)$ into the equation $y = -\dfrac{1}{2}x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 4$ into $y = -\dfrac{1}{2}x + c$.

Example 3   A line passes through the points $(0, 5)$ and $(9, -1)$.
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1 - 5}{9 - 0}$ $= \dfrac{-6}{9} = -\dfrac{2}{3}$ $-\dfrac{1}{m} = \dfrac{3}{2}$ $y = \dfrac{3}{2}x + c$ $\text{Midpoint} = \left(\dfrac{0 + 9}{2}, \dfrac{5 + (-1)}{2} \right) = \left(\dfrac{9}{2}, 2 \right)$ $2 = \dfrac{3}{2} \times \dfrac{9}{2} + c$ $c = -\dfrac{19}{4}$ $y = \dfrac{3}{2}x - \dfrac{19}{4}$	**1** Substitute the coordinates into the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** As the lines are perpendicular, the gradient of the perpendicular line is $-\dfrac{1}{m}$. **3** Substitute the gradient into the equation $y = mx + c$. **4** Work out the coordinates of the midpoint of the line. **5** Substitute the coordinates of the midpoint into the equation. **6** Simplify and solve the equation. **7** Substitute $c = -\dfrac{19}{4}$ into the equation $y = \dfrac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

   a $y = 3x + 1$ $(3, 2)$ **b** $y = 3 - 2x$ $(1, 3)$

   c $2x + 4y + 3 = 0$ $(6, -3)$ **d** $2y - 3x + 2 = 0$ $(8, 20)$

2 Find the equation of the line perpendicular to $y = \dfrac{1}{2}x - 3$ which passes through the point $(-5, 3)$.

> **Hint**
>
> If $m = \dfrac{a}{b}$ then the negative reciprocal $-\dfrac{1}{m} = -\dfrac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

   a $y = 2x - 6$ $(4, 0)$ **b** $y = -\dfrac{1}{3}x + \dfrac{1}{2}$ $(2, 13)$

   c $x - 4y - 4 = 0$ $(5, 15)$ **d** $5y + 2x - 5 = 0$ $(6, 7)$

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

    a $(4, 3)$, $(-2, -9)$ **b** $(0, 3)$, $(-10, 8)$

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

    a $y = 2x + 3$ **b** $y = 3x$ **c** $y = 4x - 3$
        $y = 2x - 7$ $2x + y - 3 = 0$ $4y + x = 2$

    d $3x - y + 5 = 0$ **e** $2x + 5y - 1 = 0$ **f** $2x - y = 6$
        $x + 3y = 1$ $y = 2x + 7$ $6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

    a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

    The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

    b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

    The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

    c Find an equation of $\mathbf{L_3}$

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\dfrac{1}{2}x$ **d** $y = \dfrac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\dfrac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \dfrac{5}{2}x - 8$

4 **a** $y = -\dfrac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel   **b** Neither   **c** Perpendicular
d Perpendicular   **e** Neither   **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$