Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1   Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq\ x\ < 4$	Divide all three terms by 4.

Example 2   Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3   Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4   Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

Example 5   Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Practice

1 Solve these inequalities.

   a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

   d $5 - 2x < 12$ **e** $\dfrac{x}{2} \geq 5$ **f** $8 < 3 - \dfrac{x}{3}$

2 Solve these inequalities.

   a $\dfrac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

   a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

   d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

   a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

   a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

1 **a** $x > 4$ **b** $x \le 2$ **c** $x \le -1$

d $x > -\dfrac{7}{2}$ **e** $x \ge 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \le 3.5$ **c** $x < 4$

3 **a** $x \le -4$ **b** $-1 \le x < 5$ **c** $x \le 1$

d $x < -3$ **e** $x > 2$ **f** $x \le -6$

4 **a** $t < \dfrac{5}{2}$ **b** $n \ge \dfrac{7}{5}$

5 **a** $x < -6$ **b** $x < \dfrac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)