

Solving linear and quadratic simultaneous equations

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$ $x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$ $2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$ <p>So $x = 2$ or $x = -3$</p> <p>Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$</p> <p>So the solutions are $x = 2, y = 3$ and $x = -3, y = -2$</p> <p>Check:</p> <p>equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES</p> <p>equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES</p>	<ol style="list-style-type: none"> 1 Substitute $x + 1$ for y into the second equation. 2 Expand the brackets and simplify. 3 Factorise the quadratic equation. 4 Work out the values of x. 5 To find the value of y, substitute both values of x into one of the original equations. 6 Substitute both pairs of values of x and y into both equations to check your answers.
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Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$ $2y^2 + \left(\frac{5-3y}{2}\right)y = 12$ $2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$ $(y+8)(y-3) = 0$ <p>So $y = -8$ or $y = 3$</p> <p>Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$</p> <p>So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$</p> <p>Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES</p>	<ol style="list-style-type: none"> 1 Rearrange the first equation. 2 Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y. 3 Expand the brackets and simplify. 4 Factorise the quadratic equation. 5 Work out the values of y. 6 To find the value of x, substitute both values of y into one of the original equations. 7 Substitute both pairs of values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
 $x^2 + y^2 = 10$

2 $y = 6 - x$
 $x^2 + y^2 = 20$

3 $y = x - 3$
 $x^2 + y^2 = 5$

4 $y = 9 - 2x$
 $x^2 + y^2 = 17$

5 $y = 3x - 5$
 $y = x^2 - 2x + 1$

6 $y = x - 5$
 $y = x^2 - 5x - 12$

7 $y = x + 5$
 $x^2 + y^2 = 25$

8 $y = 2x - 1$
 $x^2 + xy = 24$

9 $y = 2x$
 $y^2 - xy = 8$

10 $2x + y = 11$
 $xy = 15$

Extend

11 $x - y = 1$
 $x^2 + y^2 = 3$

12 $y - x = 2$
 $x^2 + xy = 3$

Answers

1 $x = 1, y = 3$

$$x = -\frac{9}{5}, y = -\frac{13}{5}$$

2 $x = 2, y = 4$

$$x = 4, y = 2$$

3 $x = 1, y = -2$

$$x = 2, y = -1$$

4 $x = 4, y = 1$

$$x = \frac{16}{5}, y = \frac{13}{5}$$

5 $x = 3, y = 4$

$$x = 2, y = 1$$

6 $x = 7, y = 2$

$$x = -1, y = -6$$

7 $x = 0, y = 5$

$$x = -5, y = 0$$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$$x = 3, y = 5$$

9 $x = -2, y = -4$

$$x = 2, y = 4$$

10 $x = \frac{5}{2}, y = 6$

$$x = 3, y = 5$$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$$