# **Completing the square**

#### A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

# **Key points**

- Completing the square for a quadratic rearranges  $ax^2 + bx + c$  into the form  $p(x+q)^2 + r$
- If  $a \neq 1$ , then factorise using a as a common factor.

# **Examples**

**Example 1** Complete the square for the quadratic expression  $x^2 + 6x - 2$ 

$$x^{2} + 6x - 2$$
=  $(x + 3)^{2} - 9 - 2$ 
=  $(x + 3)^{2} - 11$ 

1 Write  $x^{2} + bx + c$  in the form
$$\left(x + \frac{b}{2}\right)^{2} - \left(\frac{b}{2}\right)^{2} + c$$
2 Simplify

**Example 2** Write  $2x^2 - 5x + 1$  in the form  $p(x+q)^2 + r$ 

$$2x^{2} - 5x + 1$$

$$= 2\left(x^{2} - \frac{5}{2}x\right) + 1$$

$$= 2\left[\left(x - \frac{5}{4}\right)^{2} - \left(\frac{5}{4}\right)^{2}\right] + 1$$

$$= 2\left[\left(x - \frac{5}{4}\right)^{2} - \frac{25}{8} + 1\right]$$

$$= 2\left(x - \frac{5}{4}\right)^{2} - \frac{25}{8} + 1$$
3 Expand the square brackets – don't forget to multiply  $\left(\frac{5}{4}\right)^{2}$  by the factor of 2
$$= 2\left(x - \frac{5}{4}\right)^{2} - \frac{17}{8}$$
4 Simplify

## **Practice**

1 Write the following quadratic expressions in the form  $(x + p)^2 + q$ 

**a** 
$$x^2 + 4x + 3$$

**b** 
$$x^2 - 10x - 3$$

c 
$$x^2 - 8x$$

**d** 
$$x^2 + 6x$$

e 
$$x^2 - 2x + 7$$

$$\mathbf{f} = x^2 + 3x - 2$$

2 Write the following quadratic expressions in the form  $p(x+q)^2 + r$ 

a 
$$2x^2 - 8x - 16$$

**b** 
$$4x^2 - 8x - 16$$

c 
$$3x^2 + 12x - 9$$

**d** 
$$2x^2 + 6x - 8$$

**3** Complete the square.

a 
$$2x^2 + 3x + 6$$

**b** 
$$3x^2 - 2x$$

c 
$$5x^2 + 3x$$

**d** 
$$3x^2 + 5x + 3$$

#### **Extend**

4 Write  $(25x^2 + 30x + 12)$  in the form  $(ax + b)^2 + c$ .

## Answers

1 **a**  $(x+2)^2-1$ 

c  $(x-4)^2-16$ 

 $e (x-1)^2 + 6$ 

2 **a**  $2(x-2)^2-24$ 

c  $3(x+2)^2-21$ 

3 **a**  $2\left(x+\frac{3}{4}\right)^2+\frac{39}{8}$ 

 $c = 5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ 

4  $(5x+3)^2+3$ 

**b**  $(x-5)^2-28$ 

**d**  $(x+3)^2-9$ 

 $f = \left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$ 

**b**  $4(x-1)^2-20$ 

**d**  $2\left(x+\frac{3}{2}\right)^2-\frac{25}{2}$ 

**b**  $3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}$ 

**d**  $3\left(x+\frac{5}{6}\right)^2+\frac{11}{12}$